BRIDGE INSTRUMENTS

Bridge instruments have a broad field of applications in metrology and industrial laboratories, not only for the comparison or the determination of electrical quantities but also for the measurement of nonelectrical quantities by electrical methods. The aim of this article is to combine the need to provide an understanding of the principles used in some modern dc and ac bridge instruments without forgetting those involved in the most classical ones. This article is divided in three parts: bridge networks, dc bridge instruments, and ac bridge instruments.

BRIDGE NETWORKS

Elementary Dipoles

Bridge networks are realized by the connection of elementary electrical elements generally fitted with two terminals and called dipoles. An electrical dipole is an electrical element which could be externally connected by means of its two terminals. The current entering by one terminal is identical to the one leaving the other. One electrical dipole can supply or

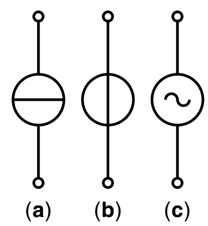

Figure 1. Representation of the used active dipoles: (a) dc current source. (b) dc voltage source. (c) ac generator.

transform electrical energy. Two main categories of dipoles can be distinguished:

- Active dipoles like voltage sources and current sources
- Passive dipoles like resistors, capacitors and inductors

If one dipole is linear, its behavior is governed by a mathematically well-defined linear correspondence between the applied voltage and the flowing current. Then, we can write for each dipole:

- A relationship voltage-current $v = f(i)$
- A relationship current-voltage $i = g(v)$

Sources. The sources are dipoles which are able to modify the electrical state of networks. There are two types of ideal sources: voltage sources and current sources. The ideal voltage source is a hypothetical dipole which maintains a constant voltage at its terminals, independent of the current flowing through it:

$$\frac{dv}{di} \equiv 0 \tag{1}$$

The ideal current source is a dipole for which the current is independent of the voltage at its terminals:

$$\frac{di}{dv} \equiv 0 \tag{2}$$

The symbols used for the dc sources are those indicated in Recommendation No. 117 of the International Electrotechnical Commission (IEC) and represented in Fig. 1(a) and 1(b). To distinguish the ac sources and due to the fact that it is very common in the literature, the symbol used has been abstracted from IEEE Standard 315-1975 and is shown in Fig. 1(c).

Passive Dipoles

Resistor. A resistor of resistance R and of conductance G is a dipole which creates a voltage drop $v(t)$ at its terminals, when flown through by an instantaneous current $i(t)$:

$$v(t) = Ri(t) \tag{3}$$

The inverse relationship current-voltage is

$$i(t) = Gv(t) \tag{4}$$

Capacitor. A perfect capacitor of capacitance C is a dipole which possesses an electrical charge $q(t)$ when the instantaneous voltage at its terminals is $v(t)$:

$$q(t) = Cv(t) \tag{5}$$

The relation of current-voltage can be written

$$i(t) = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt} \tag{6}$$

and the relationship of voltage-current:

$$v(t) = \frac{1}{C}\int_0^t i(t)\,dt + v(0) \tag{7}$$

where $v(0)$ is the value of v at $t = 0$.

Inductor. A perfect inductor of inductance L is a dipole which embraces a magnetic flux $\Phi(t)$ when flown through by an instantaneous current $i(t)$:

$$\Phi(t) = Li(t) \tag{8}$$

The instantaneous voltage at the terminals of this inductor is

$$v(t) = \frac{d\Phi(t)}{dt} = L\frac{di(t)}{dt} \tag{9}$$

The relationship of current-voltage is

$$i(t) = \frac{1}{L}\int_0^t i(t)\,dt + v(0) \tag{10}$$

where $i(0)$ is the value of i at $t = 0$.

Passive Dipoles in ac

If we apply an ac voltage to each of the previous passive dipoles, an ac current $i(t) = I_M \cos\omega t$ will flow through them. We will hereafter successively examine each dipole.

Resistor. The voltage difference at the terminals of the resistor is

$$v_R = RI_M \cos\omega t = V_M \cos\omega t \tag{11}$$

The impedance Z and the admittance Y are real numbers:

$$Z = R \quad \text{and} \quad Y = \frac{1}{R} \tag{12}$$

Inductor. The voltage difference at the terminals of the inductor is

$$v_L = L\frac{di}{dt} = -L\omega I_M \sin\omega t = L\omega I_M \cos\left(\omega t + \frac{\pi}{2}\right) \tag{13}$$

The impedance Z and the admittance Y are imaginary numbers:

$$Z = jL\omega \quad \text{and} \quad Y = \frac{1}{j\omega L} \tag{14}$$

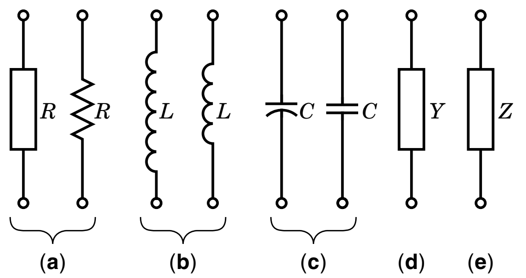

Figure 2. Representation of the used passive dipoles: (a) Resistor. (b) Inductor. (c) Capacitor. (d) Admittance. (e) Impedance.

Capacitor. The voltage difference at the terminals of the inductor is

$$v_C = \frac{1}{C}\int_0^t i dt + v_C(0) \tag{15}$$

if $v_C(0) = 0$ at $t = 0$

Then, we have

$$v_C = \frac{I_M}{C\omega}\sin\omega t = \frac{I_M}{C\omega}\cos\left(\omega t - \frac{\pi}{2}\right) \tag{16}$$

The impedance Z and admittance Y are also imaginary numbers:

$$Z = \frac{1}{j\omega C} \quad \text{and} \quad Y = j\omega C \tag{17}$$

Symbols. In this document, the symbols used to represent the different passive dipoles are coming from the same IEEE Standard or IEC Recommendation as already indicated previously and are shown in Fig. 2(a–e).

Lumped Networks in ac

A lumped network is realized by the connection of separate passive dipoles like a resistor, an inductor, and a capacitor. This concept is very familiar in ac bridges and is used for representation of imperfect dipoles. An imperfect inductor is represented in two ways in Fig. 3(a) and (b). The conversion relationships between the two representations are

$$L_S = L_P(1 + Q^{-2}) \quad \text{and} \quad R_S = \frac{R_P}{1 + Q^2} \tag{18}$$

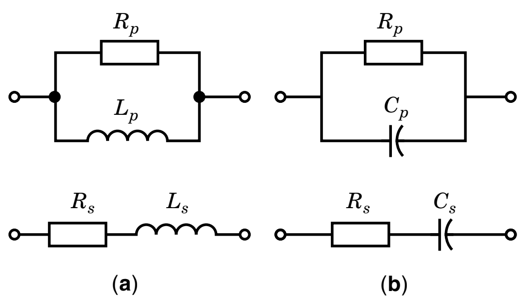

Figure 3. Lumped networks in ac: (a) Inductive (b) Capacitive.

where

$$Q = \frac{R_P}{\omega L_P} = \frac{\omega L_S}{R_S} = \frac{1}{D} = \frac{1}{\tan\delta} \tag{19}$$

Similarly, for an imperfect capacitor the relationships are

$$C_s = C_p(1 + D^2) \quad \text{and} \quad R_S = \frac{R_P}{1 + D^{-2}} \tag{20}$$

where

$$D = \frac{1}{\omega C_P R_P} = \omega C_S R_S = \tan\delta = \frac{1}{Q} \tag{21}$$

Generally, the series representation is preferred for inductors, and the parallel one is preferred for capacitors.

Kirchhoff's Laws

Two laws due to Kirchhoff form the basis of the electrical network theory.

Kirchhoff's Current Law. The current law is applicable to any node and may be expressed as

$$\sum i(t) = 0 \tag{22}$$

This law may be regarded as a principle of conservation of charge.

Kirchhoff's Voltage Law. The voltage law is applicable to any loop, is made up of a set of electrical dipoles connected end to end in order to form a closed circuit, and may be expressed, in the case of purely resistive dipoles, as:

$$\sum e(t) = \sum Ri(t) \tag{23}$$

That is a generalization of Ohm's law.

General Principles Applicable to Linear Electrical Networks

An electrical network consists of a set of electrical dipoles connected together. If an electrical network is externally accessible by only two terminals, it constitutes by itself a dipole. An electrical network which is accessible from the external by means of four terminals, is considered like a quadripole. The general case is a multipole. We will only examine the principles which are useful for the understanding of the following sections. Indeed, complementary explanations about these principles can also be found in this encyclopedia or in other specialized books.

It is also assumed that the examined networks are linear. This means, in particular, that the current in any branch of a network caused by a source located in the same or any other branch is proportional to the electromotive force of that source. A branch is made up of one electrical dipole or of more electrical dipoles connected in series and flown through by the same current.

Superposition Principle (Helmholtz's Theorem). The current in any branch of a network is the sum of the currents which would be induced in the branch by each source of the network when all other sources are either short-circuited (voltage source) or open (current source).

Duality Principle. The current-voltage relationships for a given network can be deduced from the voltage–current relationships of the reciprocal network by application of the following bijections:

$$\begin{aligned} &v \Leftrightarrow i \\ &R \Leftrightarrow G \\ &L \Leftrightarrow C \\ &\phi \Leftrightarrow q \\ &\text{series} \Leftrightarrow \text{parallel} \\ &\text{loop} \Leftrightarrow \text{node} \\ &\frac{dv}{di} \equiv 0 \Leftrightarrow \frac{di}{dv} \equiv 0 \\ &v \equiv 0 \Leftrightarrow i \equiv 0 \end{aligned} \tag{24}$$

This principle is useful when it is requested to solve a problem where the solution of the reciprocal problem is already known. So, in an electrical bridge, by application of this principle, the source and the detector may be interchanged without altering the balance condition. However, this change also modifies the impedance matching between the source and the detector.

DC BRIDGE INSTRUMENTS

Definitions for a dc Resistor

The value of the combined standard measurement uncertainty, which accompanies the measurement result of a resistor, is often dependent of the characteristics of the device, itself. The factors contributing to this uncertainty are mainly

- Imperfect design of the resistor
- Connection terminals and the insulation materials
- Influence of the measurement parameters such as the temperature, the relative humidity, the pressure, and the measuring current.

At present, the most completely defined resistor is the one called 5 Terminal. The schematic diagram of a 5 Terminal resistor is shown in Fig. 4:

- P_1 and P_2 are the potential terminals
- C_1 and C_2 are the current terminals
- G is the guard terminal and is connected to the metal container surrounding the resistive element
- R is the resistor to be measured. It must respond to the accepted definition of a four terminal resistor, which is the ratio of

$$R = \frac{U_{12}}{I_{12}} \tag{25}$$

  where U_{12} is the open circuit voltage between nodes 1 and 2, and I_{12} is the current flowing through R.
- r_{c1}, r_{c2}, r_{p1}, and r_{p2} represent the lead and terminal resistances, respectively, at the terminals C_1, C_2, P_1, and P_2.

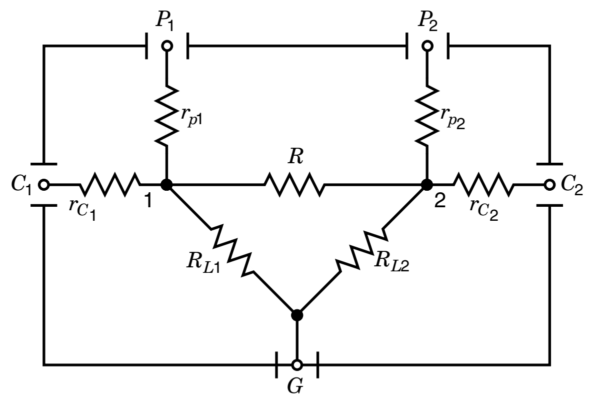

Figure 4. An equivalent circuit of a five-terminal resistor.

- R_{L1} and R_{L2} represent the leakage resistances from the same terminals C_1, C_2, P_1, and P_2.

This type of 5 Terminal resistor is now commercially available in the range from 1 Ω to 10^9 Ω.

The 4 Terminal configuration is the same as in Fig. 4 without the Guard terminal G. The measured resistance differs from the four terminal definition and is given by:

$$R_{\text{meas}} \cong R\left(1 - \frac{R}{R_{L1} + R_{L2}}\right) \tag{26}$$

From this formula, it can be seen that this design is valid for lower value dc resistors ($<10^3$ Ω). Although an old design, this is still used for the most stable dc standard resistors at the 1 Ω level (1).

In the 3 Terminal configuration, there is no distinction made between the potential and the current terminals but the guard terminal is kept. The lead and terminal resistance r_{c1} and r_{c2} for this kind of resistor add to R. The measured resistance is now given by:

$$R_{\text{meas}} = R + r_{c1} + r_{c2} \tag{27}$$

Because r_{c1} and r_{c2} for this kind of resistor are below 1 Ω, it is generally accepted that this design can be used for resistors above 10^6 Ω.

The 2 Terminal configuration is a simplified model of the 3 Terminal one. That was the first configuration used. It is still used in practice when it is compatible with the required measurement uncertainty.

Wheatstone Bridge

The bridge principle was first introduced by S. H. Christie in 1833, but was neglected until 1843, when Sir Charles Wheatstone applied this previously described arrangement of wires to the comparison of resistances (2–4).

In the original form of the Wheatstone bridge, shown in Fig. 5, X is the unknown resistor, A and B are equal resistors, and S is an adjustable resistor. S is varied until the detector deflection is zero. A battery is connected between nodes 1 and 3, and a detector joins the nodes 2 and 4. A node is a connecting point for one or more branches. S is adjusted until no potential difference exists between the junction points 2 and

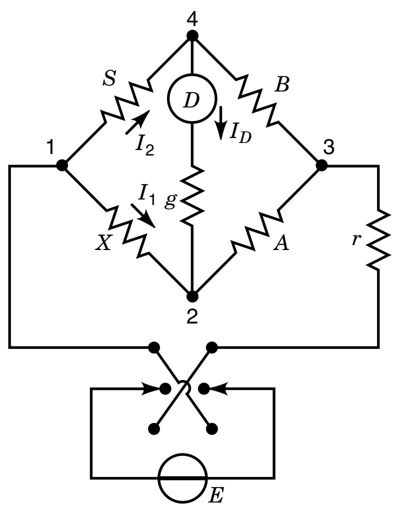

Figure 5. A Wheatstone bridge.

4, so there is no current through the detector. The magnitude of X is to be calculated from the equation:

$$X = S\frac{A}{B} \tag{28}$$

In this bridge, four of the six branches of the network containing the resistors A, B, S, and X are also called arms. Due to the fact that the resistors X and S are compared, Wheatstone called this arrangement a resistance balance. The resistors A and B are forming the arms of the balance by analogy with a weighing process.

In practice, the ratio A/B can be 1, 0.1, 0.01, or even lower. Inverse ratios 10, 100, or even higher can also be used. As also indicated in Fig. 5, the polarity of the voltage source is periodically reversed to reduce the effect of thermoelectric voltages. Thermoelectric voltages are mainly generated at the junctions of dissimilar metals at different temperatures. If we consider that the voltage source E has a negligible internal resistance r, and that the null detector has an input resistance g, we can calculate the current I_D in the detector by solving three loop equations in the networks *123E, 143E,* and *124* as follows:

$$\begin{bmatrix} X+A & 0 & -X \\ 0 & S+B & S \\ -X & S & S+g+X \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_D \end{bmatrix} = \begin{bmatrix} E \\ E \\ 0 \end{bmatrix}$$

$$I_D = \frac{\begin{vmatrix} X+A & 0 & E \\ 0 & S+B & E \\ -X & S & 0 \end{vmatrix}}{\begin{vmatrix} X+A & 0 & -X \\ 0 & S+B & S \\ -X & S & S+g+X \end{vmatrix}}$$

$$I_D = E\frac{X(S+B) - S(X+A)}{(X+A)(S+B)(S+g+X) - X^2(S+B) - S^2(X+A)}$$

$$I_D = E\frac{XB - SA}{AX(B+S) + BS(A+X) + g(A+X)(B+S)} \tag{29}$$

When the bridge is balanced, $I_D = 0$, and we find the same relation as in the Eq. (28).

The first two products of the denominator may be replaced by

$$AB(S+X) + SX(A+B)$$

or by

$$AS(B+X) + BX(A+S)$$

The relative current sensitivity σ_i of the Wheatstone bridge near the balance condition can be expressed as follows:

$$|\sigma_i| = \left|\frac{\Delta I_D}{\frac{\Delta S}{S}}\right|_{\Delta S \to 0} \tag{30}$$

The sensitivity σ_i of a bridge can be optimized by selecting the elements of the bridge. If we slightly modify the balance of the Wheatstone bridge given in Fig. 5 by changing the adjustable resistor S by a small amount ΔS, the corresponding variation ΔI_D indicated by the detector will be given according to Eq. (29):

$$|\Delta_{I_D}| = \left|E\frac{A\Delta S}{AX(B+S) + BS(A+X) + g(A+X)(B+S)}\right| \tag{31}$$

knowing that at balance, we have also

$$\frac{A}{B} = \frac{X}{S} = \frac{A+X}{B+S} \tag{32}$$

Equation (31) can be rewritten:

$$|\Delta I_D| = \left|E\frac{A\Delta S}{AS(A+X) + BS(A+X) + gS(A+X) + gB(A+X)}\right| \tag{33}$$

$$|\Delta I_D| = \left|E\frac{A\Delta S}{(A+X)[AS + BS + gS + gB)]}\right| \tag{34}$$

$$|\sigma_i| = \left|\frac{\Delta I_D}{\frac{\Delta S}{S}}\right| = \left|E\frac{A}{(A+X)\left[A + B + g + \frac{gB}{S}\right]}\right| \tag{35}$$

The parameters of the bridge must be chosen so that this current sensitivity is maximum, taking into account that the value of the source voltage E is limited by the admissible power dissipation in each resistor.

The usual Wheatstone bridge is designed for the measurement of 2 Terminal devices and so is not suitable for accurate resistance measurements below 1 MΩ.

At present, the Wheatstone bridge remains the basic bridge for many industrial applications with sensors like strain-gauge installations for force, torque, or pressure measurements.

Automated Guarded Wheatstone Bridge

At present in metrology laboratories, the main application of the Wheatstone bridge is reserved for the measurement of high value resistors.

In this case, a guarded Wheatstone bridge is used to compare 3 Terminal resistors. The availability of highly linear

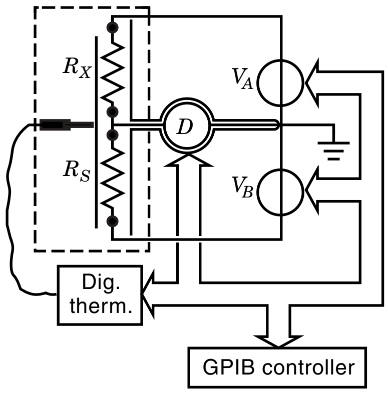

Figure 6. An automated guarded Wheatstone bridge.

and fully computer-controlled dc sources or calibrators based on the pulse width modulation technique has led to the development of a new type of automated guarded Wheatstone bridge represented in Fig. 6 (5–7). In this bridge, the arm resistors A and B are advantageously replaced by two dc calibrators V_A and V_B having a very good linearity, a low temperature coefficient, and low output impedance. This bridge can be balanced automatically. At balance, the value of R_x is given by the following equation:

$$R_x = \frac{V_A}{V_B} R_S \tag{36}$$

This type of bridge can be used to calibrate resistors from 10 MΩ up to 1 TΩ or more in less time and with lower uncertainty than with the conventional guarded Wheatstone bridge. Combined measurement uncertainties of order of 1 part in 10^5 can be obtained at 1 GΩ with this bridge.

Kelvin Double Bridge

William Thomson (later Lord Kelvin) first described this bridge in a paper published in 1862. This network became known as the Thomson bridge, the Thomson double bridge, the Kelvin bridge, and the Kelvin double bridge. According to F. Wenner (2), Thomson seems to have been the first to attempt measurements of the highest precision attainable with the apparatus then available and to have an understanding of the factors limiting the precision of measurement. As shown in the schematic diagram of Fig. 7, the circuit contains a second set of ratio arms, labeled a and b. The guard circuit is not represented in Fig. 7. This Kelvin bridge may be transformed into an equivalent Wheatstone bridge. The Δ network formed by arms a and b and by link k can indeed be transformed in the Y network by using the theorem of Kennelly.

The detector indication will be zero when the voltage at node 2 equals the voltage at node 5.

$$U_2 = \frac{A}{A+B} I \left[S + X + \frac{(a+b)k}{a+b+k} \right] \tag{37}$$

$$U_5 = \mathrm{I} \left\{ S + \frac{a}{a+b} \left[\frac{(a+b)k}{a+b+k} \right] \right\} \tag{38}$$

We can solve for X by equating U_2 and U_5 and simplifying:

$$S + X + \frac{(a+b)k}{a+b+k} = \frac{A+B}{A} \left\{ S + \frac{a}{a+b} \left[\frac{(a+b)k}{a+b+k} \right] \right\} \tag{39}$$

$$X = S\frac{B}{A} + \frac{ak}{a+b+k} \left(\frac{B}{A} - \frac{b}{a} \right) \tag{40}$$

By making two successive balances using configurations with and without the link k opened, the condition $B/A = b/a$ is fulfilled. The well-known relationship is established:

$$X = S\frac{B}{A} \tag{41}$$

This bridge is suitable for comparing 5 Terminal and 4 Terminal resistors. Nevertheless, it does not entirely satisfy the theoretical definition of a four terminal resistor. Indeed, there exists a voltage difference between nodes 1 and 3. This means, in particular, that a current is flowing through one of the potential leads and terminals of each resistor X and S. This bridge is especially useful for the measurement of 4 Terminal resistors below 100 Ω.

Warshawsky Bridge

I. Warshawsky described this bridge in 1955 (8). The schematic diagram is given in Fig. 8. The screens of the resistors are not represented on this diagram. This bridge is of the Wheatstone type and is fitted at each of its four corners with a combining network. Each combining network consists of two auxiliary resistors connecting terminals of adjacent main resistor and also consists of one low-resistance link joining the other terminals. At each corner, the ratio of the auxiliary resistors must be the same as that of the main resistors they join. The exact balance equation of this bridge can be calculated by transforming, at each corner, the Δ network in a T network. The ideal relationship for this bridge is similar to that of the Wheatstone bridge:

$$X = \frac{B}{A} S \tag{42}$$

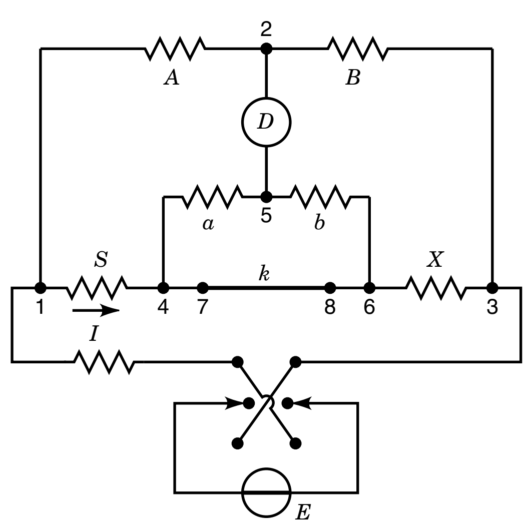

Figure 7. A Kelvin double bridge.

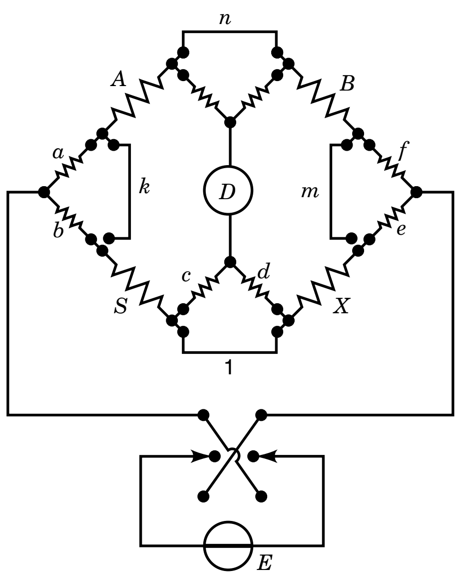

Figure 8. A Warshawsky bridge.

Departure of less than 1 part in 10^8 from this ideal relationship has been reported if all the resistors of the bridge satisfy the following conditions (9):

- The four main resistors are within 1 part in 10^4 of their mean value.
- The auxiliary resistors are matched pairwise to better than 1 part in 10^4.
- The low-resistance links are less than 1 part in 10^4 of the main and auxiliary resistances.

These conditions can be achieved in particular when main resistors of 10 kΩ are compared.

Dc Current Comparator Bridge

Originally, the basic design of this bridge comes from the work of N. L. Kusters et al. at the National Research Council of Canada (10,11). The essential feature of the dc current comparator bridge (CCB) is shown in Fig. 9. This CCB consists of

1. Two uniformly distributed ratio windings of N_x and N_s turns, wound on a toroidal magnetic core, are excited in a opposite way by two isolated and reversible direct current sources acting as Master and Slave
2. A zero flux detector and a double core magnetic modulator with a magnetic shield. This magnetic shield has the form of a hollow toroid, made of high permeability material
3. A null detector

The balancing of the bridge is completed when two balance conditions are fulfilled:

- The magnetomotive force balance is achieved automatically by means of the servo circuit operating from the output signal of the zero flux detector. This servo circuit adjusts the current I_s delivered by the slave power supply in order to maintain the difference of the magnetomotive forces of the two windings as close to zero as possible. This gives the first condition:

$$N_s I_s = N_x I_x \tag{43}$$

- The zero voltage condition which is obtained, in the manually operated version of this type of bridge, by changing the turns ratio N_x by rotating switches until the zero indication of the null detector is achieved. This gives the second condition:

$$I_x R_s = I_x R_x \tag{44}$$

From these above-mentioned relations, it can be deduced

$$R_x = R_s \frac{N_s}{N_x} \tag{45}$$

The dc current comparator has several advantages with respect to the previously described dc bridges:

1. The measured ratio depends for the most significant decades only on numbers of turns. This means that this type of bridge are not likely to drift with time or to change with ambient temperature, relative humidity, and atmospheric pressure.
2. The latest available versions of this kind of current comparator bridge are automatic and feature a linearity often better than 3 parts in 10^8.
3. It is possible to compare resistors according to the 4 Terminal and 5 Terminal definitions.
4. The unknown and standard resistors are not connected in series. This means that the current flowing through them can be different.

In addition to the limitations common to other bridges due to the resolution of the null detector and thermal noise, we have two other sources of noise: Barkhausen noise in the magnetic

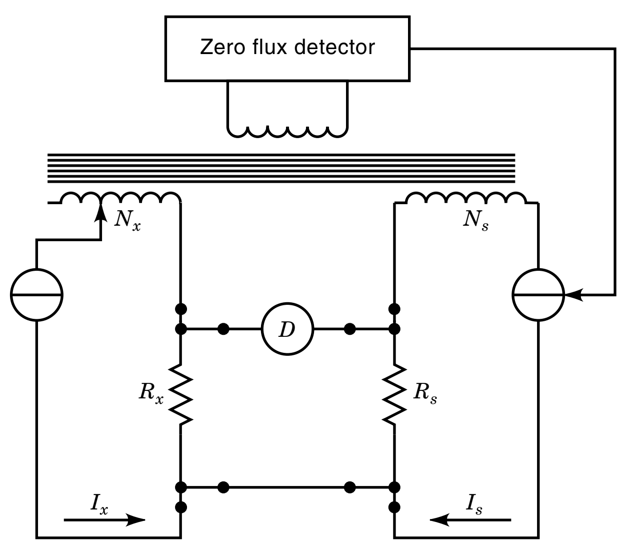

Figure 9. A dc current comparator bridge.

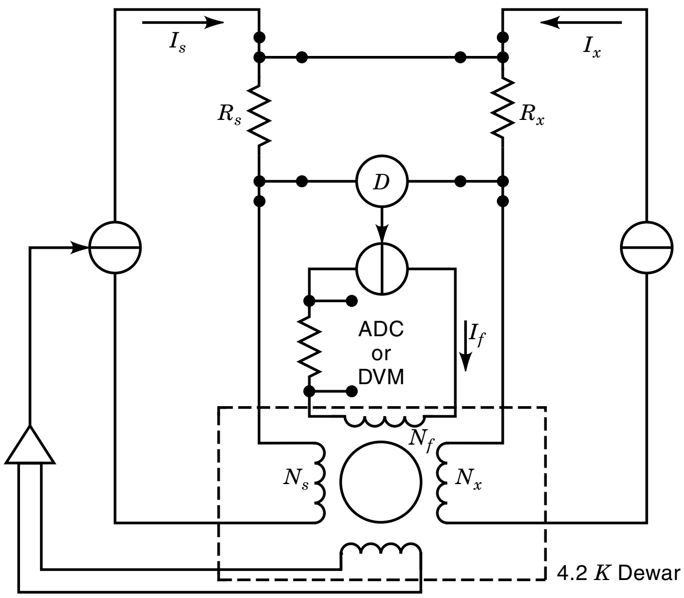

Figure 10. A dc cryogenic current comparator bridge.

core and electrical noise in the electronic servo circuit. Currently, commercial versions of this type of dc current comparator are automatic. They can be used to measure dc resistors and shunts between 1 mΩ and 1 MΩ. Their sensitivity is about 0.1 μA for a winding of 1 turn.

Dc Cryogenic Current Comparator Bridge

Similar in its principle to the dc CCB, the dc cryogenic CCB has its ratio windings, toroidal overlapping lead shield, detection winding and flux detector made of superconductors or superconducting devices. As for the dc current comparator bridge, at balance, the ratio of currents I_s and I_x flowing through the windings is also the inverse ratio of the number of turns. The working principle is the Meissner effect. If the magnetomotive forces generated by the windings inside the superconducting shield are not fully equal and in opposition, a current, proportional to this imbalance, will circulate in the outer surface of this toroidal shield. This current will be detected by means of a superconducting flux transformer made of a low inductance superconducting coil. The latter is connected in series with the superconducting input coil of a very sensitive SQUID (Superconducting Quantum Interference Device) magnetometer. This type of magnetometer is presently the most sensitive magnetic field sensor known. A schematic diagram of a cryogenic current comparator bridge (CCCB) is illustrated in Fig. 10. Two reversible isolated and fully floating current sources generate currents I_s and I_x in approximately the inverse ratio of the winding turns N_s and N_x. The SQUID provides the magnetomotive force balance by controlling the Slave current source through the SQUID output and an isolation amplifier. The null detector amplifies the potential difference across the two resistors R_s and R_x. The output of this detector is used to drive a third current source, which provides a feedback current I_f through a feedback winding N_f. This feedback current is known by measuring the voltage across a known resistor with a digital voltmeter or an analogue to digital converter (ADC). At balance, we have again two equations. At zero indication of the null detector, we have

$$R_s I_s = R_x I_x \tag{46}$$

and when the magnetomotive force balance is achieved,

$$N_x I_x + I_f N_f - N_s I_s = 0 \tag{47}$$

If we replace I_s in (46), we obtain

$$\frac{R_x}{R_s} = \frac{N_x}{N_s}\left(1 + \frac{I_f N_f}{I_x N_x}\right) \tag{48}$$

A typical 1 σ combined uncertainty of a few parts in 10^9 can be obtained with a commercially available CCCB (12) in the measurement of 1 : 1 ratios for resistors of 100 Ω. This uncertainty depends mainly on

- Calibration of the bridge and accessories
- Leakage resistances
- Electrical noise in the null detector, the SQUID, and the resistors

A sensitivity of about 1 nA for a winding of one turn can be achieved.

Dc Sources

The balance conditions calculated above for most of the bridges are independent of the values of the voltage sources. To reduce the influence of thermal emf's, the polarity of the source is regularly and often automatically reversed. It is reasonable to think that after these reversal operations, the voltage of the source must be kept reasonably stable.

The advantages of computer controlled dc calibrators have also been demonstrated for the realization of automated guarded Wheatstone bridges. In the past, depending on the value of the measuring current, batteries or lead acid accumulators were generally used as sources in dc bridges. Mercury batteries were also often used in different bridges because of their low drift and low temperature coefficient. However, they are no longer available because of new legislation concerning the environment. Today, the efficiency of shielding and noise reduction techniques has also increased, so those sources operating from the electrical mains are now more often used. Also, thanks to design improvements and automatization, power supplies, sometimes connected in parallel, are now the most convenient solution for the calibration with dc current comparator bridges of large current-carrying resistors or resistive shunts for currents from 1 A to 2000 A or more. For applications involving low power consumption and when it is strictly necessary to avoid noise and leakage paths due to the electrical mains, batteries can provide fully isolated voltage or current sources.

Dc Detectors

Today, electronic null detectors have replaced nearly all the galvanometers and the photogalvanometer amplifiers. Some electronic null detectors, nanovoltmeters, and certainly picovoltmeters exceed the sensitivity of most of the older detectors. Picovoltmeters operating at 4.2 K and based on a SQUID have been developed (13). An increasing number of these electronic instruments are more and more often operated from controlling computers via a parallel or a serial bus despite the problem arising from the additional noise introduced by this type of connection.

Noise coming from the bridge influences the null detector. This includes

- Radiated and conducted electromagnetic interference
- Thermal noise or Johnson noise due to the bridge. This means, in particular, that in some applications, the resistance of the connecting leads must be taken into account
- Barkhausen noise, which can be present with magnetic circuits like those in dc current comparators bridges

In addition to external noise and even when they are operating on internal batteries, electrical noise is also generated inside these electronic null detectors (14):

- Thermal noise or Johnson noise which emanates from all resistive devices. The value of the open-circuit voltage is given by the following equation:

$$V = \sqrt{4kTRB} \tag{49}$$

  where k = Boltzmann's constant (about 1.38×10^{-10} J/K); T = Absolute temperature in kelvin; R = Resistance in ohm; B = Equivalent noise bandwidth in hertz.
- Shot noise caused by the current flow across a potential barrier
- Popcorn noise mainly due to some manufacturing processes of semiconductors and integrated circuits
- Contact noise due to imperfect contacts in switches or relays

An equivalent circuit of a dc detector can be represented by an ideal noiseless detector of infinite input impedance, shunted by a ideal noise current generator i_n and having an ideal noise voltage source e_n in series with.

Reliable dc nanovoltmeters and picovoltmeters are now commercially available. The most sensitive of them, operating at room temperature, has an equivalent noise resistance of about 30 mΩ, a common mode rejection ratio above 180 dB, and can detect potential differences as small as a few tens of picovolts when connected to a low-impedance circuit.

Checking of dc Bridges Accuracy

To comply with international quality standards, laboratories send, at specified intervals, standard resistors and even bridges to a national metrology service or to a qualified accredited laboratory for calibration. Between such calibrations, it is good practice that each laboratory performs elementary checks on its bridges. Some checks of dc resistance bridges include

1. *Zero Voltage of the Bridge Source.* When possible, it is always useful to disconnect the power source from the bridge and to short the input leads to the bridge or to turn the output voltage down to zero. After doing this, any deflection of the null detector will be an indication of the remaining thermal emf's, external signals, and noise, which are not due to the source.
2. *Zero Resistance.* Replace the device under test by a short circuit.
3. *Infinite Resistance.* Replace the device under test by an open circuit.
4. *Checks of the Bridge Linearity.* Linearity check can be made using individual resistors or resistor networks.
   - *With Individual Resistors.* Let us assume that we have two similar resistors A and B to compare and that we make two successive measurements $Rd_1 = A/B$ and $Rd_2 = B/A$. If the bridge under test has no linearity error, the product of these readings must be equal to 1. It is interesting to point out that this check does not require knowing of the exact value of these resistors A and B. The only technical requirement for these resistors is a good short term stability.
   - *With Resistive Networks.* A network of n nominally equal four-terminal resistors connected end to end with ideal zero-resistance junctions. In this case, each resistor of the network is successively compared by means of the bridge under test with a separate one of the same nominal value. A first approximation of the linearity of the bridge at one point of the string can be obtained if the reading is equal to the sum of the individual readings up to this point:

$$R_i = \sum_{n=1}^{i} R_n$$

   - *A Hamon Resistor Network.* A Hamon network is realized by connecting first in series and then in parallel the same number of n nominally equal four terminal resistors. It can be demonstrated that if all of these resistors are within 1 in 10^4 of their mean value, the ratio of the series/parallel combination R_s/R_p is equal to n^2 within 1 part in 10^8 (15). Accurate 1/100 ratios and even 1/10 ratios can be made with these Hamon build-up resistors.
   - Very recently a resistance network, similar to the Hamon's design, was especially built for checking dc resistance bridges. It consists of a network of four main resistors connected to a common carefully engineered four-terminal junction. This produces 35 different resistance values of the four-terminal type and provides a convenient way to verify the accuracy of resistance bridges (16).

AC BRIDGE INSTRUMENTS

Many of the ac bridge instruments described in this section are mainly used in the audio-frequency range. However, some of them are still correctly operating, with higher uncertainties, slightly above the 1 MHz range.

Ac Networks

Most of the existing circuit components and even most of the impedance standards to be measured with ac bridges are nei-

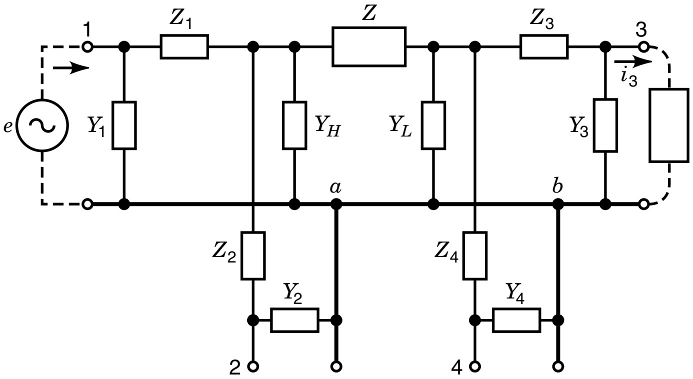

Figure 11. Four terminal-pair definition.

ther purely resistive nor purely reactive. This means that unwanted capacitance and inductance are present in resistors, unwanted resistance and inductance in capacitors as well as unwanted resistance and capacitance in inductors. The energy flow at sufficiently low frequencies in these real-world devices must be treated like a flow in a *RLC* network. When driven with an ac voltage, each *RLC* network will dissipate heat and store the energy of the generated magnetic and electric fields.

If no precaution is taken in the construction and the use of this *RLC* network, the magnetic and electric fields generated by this network will be modified by any change with respect to its immediate environment. The final measured value of this *RLC* network will be altered. As in ac bridges, the comparison is made in the energy flow through two or more sets of impedance standards; special care in order to limit the influence of these fields is the first criterion to be fulfilled in the design of these impedance standards. In theory, for a standard of capacitance, the only energy should be that associated with the electrical field *E*, and for a standard of inductance that associated with the magnetic field *H*. For a standard of resistance, the only energy should be dissipated in heat.

Ac Measured Devices

The definitions given for dc resistors are, even for the most sophisticated of them, not satisfactory in low-frequency ac measurements mainly due to the frequency dependence of the measured value, the series impedances, the shunt admittances, and the stray fields present in each device. Other definitions for the devices to be measured in ac have been developed in order to overcome these problems.

Four Terminal-Pair Definition. Today, it is generally accepted that the four terminal-pair definition is the most accurate. The interest of this definition has been firstly pointed out by R. D. Cutkosky (17) and more recently refined and intensively explained by B. P. Kibble and G. H. Rayner (18). An example of a simplified circuit of a four terminal-pair impedance is shown in Fig. 11. The numbering of the terminal-pairs or ports has been kept similar to that used by the first pioneers in this field. In order to fulfill the astatic condition, the coaxial geometry is intensively used. This means also that the impedance Z is contained in a shielded box, not represented in Fig. 11, and is connected to the measuring bridge by means of suitable coaxial sockets or plugs. In Fig. 11, again by analogy with the convention (9,18), the connections between the inner conductor of each coaxial connector are represented by a fine line and the connections between the outer conductor by a thicker line. This difference points out the low series impedance of this outer conductor per unit length. A generator may be connected to the current terminal-pair 1 to supply the necessary current and a second four terminal-pair impedance to be compared with the first one at the other current terminal-pair 3. The four terminal-pair impedance Z' is defined as

$$Z' = \frac{e_2}{i_3} \tag{50}$$

The required conditions for this definition are the following:

- At port 4: $e_4 = 0$ and $i_4 = 0$
- At port 3: $i_{3\,\mathrm{OUT}} = i_{3\,\mathrm{IN}}$
- At port 2: $i_2 = 0$

From Fig. 11, the impedance Z, including the low series impedance between the nodes a and b, can be expressed as

$$Z = Z'(1 + Z_2Y_2)(1 + Z_3Y_3) \tag{51}$$

These first order correcting terms Z_iY_i are due to the connections, the coaxial connectors, and the connecting cables. These terms may be forgotten if it is accepted that the impedance is defined at the end of the connecting cables. It can also be calculated that these correcting terms Z_iY_i are lower than 1×10^{-8} at 1 kHz but nearly reach 1×10^{-3} at 1 MHz for 1 m length of 50 Ω ordinary PTFE isolated coaxial cable. It is also worth mentioning that these correcting terms do not involve the value of the impedance Z. This means that the four terminal-pair definition is valid for all values of the impedance Z and especially as long as the ratio between the length of the connecting leads or coaxial cables and the wavelength of the applied signal remains negligible. In practice, achievement of the definition condition at port 4 is limited by the sensitivity of the used null detector and usually requires the use of a combining network connected to both impedances to be compared. At port 3, the equality of the inward and outward currents at the terminals is obtained by means of a current equalizer. Port 2 is connected to a reference output voltage potential of the bridge, and auxiliary adjustments are necessary until no current flows at port 2. Despite the complexity of bridges designed for the calibration of four terminal-pair impedances, they constitute today the most widely accepted way to achieve the lowest uncertainty.

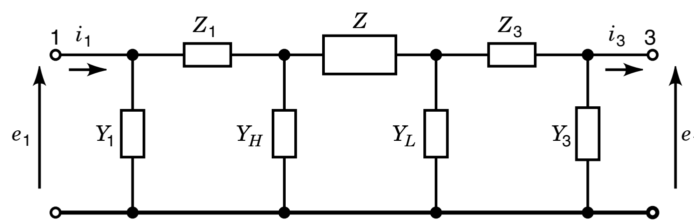

Figure 12. Two terminal-pair definition.

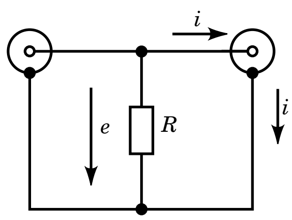

Figure 13. Four terminal coaxial definition.

Two Terminal-Pair Definition. This definition is illustrated in Fig. 12. In comparison with Fig. 11, ports 2 and 4 have been deleted. The two terminal-pair impedance is defined as

$$Z' = \frac{e_1}{i_3} \tag{52}$$

The required condition is the following:

- At port 3: $e_3 = 0$ and $i_{3\,\mathrm{OUT}} = i_{3\,\mathrm{IN}}$

By analogy with the four terminal-pair definition, we have

$$Z' = kZ \tag{53}$$

where k is a correcting factor, the value of which can be found in the literature (17,18). From a practical point of view, it can be seen from Fig. 12 that the values of Y_{H} and Y_{L} do not affect this definition. However, Y_{H} and Y_{L} enter in the correcting factor k.

Four-Terminal Coaxial Definition. A simplified circuit illustrating this definition is shown in Fig. 13. The leakage resistance, the capacitance, the series resistance, and inductance of the connecting cables and of the input connectors, which are not represented in this figure, are also influencing the measured device. This type of definition is mainly suitable for resistive devices. Several ac resistance thermometer bridges are based on this definition (19).

Three-Terminal Definition. Many capacitors manufactured in the past were based on this definition. In some of them, the outer conductor of the high potential connector is not connected to the outer conductor of the low potential one. This is not a fundamental problem when they are properly used.

Four-Arm ac Bridges

In ac, the conventional four-arm resistive bridges (also called Wheatstone bridges) and more complicated multi-arm bridges are used. Generally, these multi-arm bridges are modified four-arm bridges and can, in principle, be reduced to simple Wheatstone bridges through $Y - \Delta$ or $\Delta - Y$ transformations (theorem of Kennelly). The problem becomes more difficult when the arms of these bridges consist of complex impedances as it is the case for the measurement of inductors, and capacitors. A complex impedance can be arranged like a network in which the resistors, the inductors and capacitors can be connected in series or in parallel or both. This means that there exist a great variety of four-arm ac bridges. This multiplicity of possible network arrangements makes the classification of ac bridges somehow difficult taking also into account that some combinations of these networks are impractical and even impossible to balance.

The most general form of a four-arm ac impedance bridge is the generalization in ac of the dc Wheatstone bridge, represented in Fig. 14. The structure is similar but the resistances R_1, R_2, R_3, R_4 are replaced by impedances Z_1, Z_2, Z_3, Z_4. The generator G and detector D are operating at least partially in a common frequency range. At balance, the general equation of this bridge will be

$$Z_1 Z_3 = Z_2 Z_4 \tag{54}$$

Using the complex notation and substituting in Eq. (54)

$$(R_1 + jX_1)(R_3 + jX_3) = (R_2 + jX_2)(R_4 + jX_4) \tag{55}$$

By solving the real and imaginary terms separately, we obtain

$$R_1 R_3 - X_1 X_3 = R_2 R_4 - X_2 X_4 \tag{56}$$

$$X_1 R_3 + X_3 R_1 = X_2 R_4 + X_4 R_2 \tag{57}$$

The main difference with the dc version is that the balance condition is given by a system of two equations which must be simultaneously fulfilled. This is rather normal because two unknown quantities, which are the resistive and reactive components of the same complex impedance, have to be measured. The balance calls for the simultaneous adjustment of at least one resistive and one reactive parameter in the network. This becomes clearer if we rewrite the general balance equation in the form:

$$|Z_1 Z_3| e^{j(\varphi_1 + \varphi_3)} = |Z_2 Z_4| e^{j(\varphi_2 + \varphi_4)} \tag{58}$$

The two conditions for the balance of an ac bridge in magnitude and in phase can then be expressed as follows:

$$|Z_1 Z_3| = |Z_2 Z_4| \tag{59}$$

$$\varphi_1 + \varphi_3 = \varphi_2 + \varphi_4 \tag{60}$$

Thus, the balance of an ac bridge is obtained when the products of the magnitudes and the sum of the phase angles of the impedances in the pairwise opposite arms are made equal.

We will only consider the most useful of these bridges. To simplify the study of the different bridges, the balance condi-

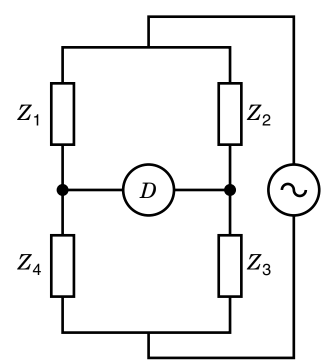

Figure 14. General form of a four-arm impedance bridge.

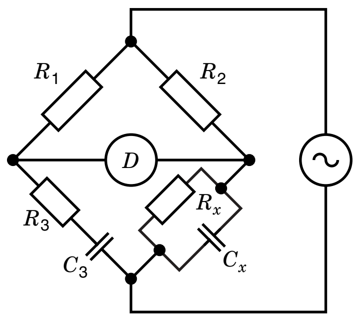

Figure 15. A Wien capacitance bridge.

tions of each bridge will be determined assuming that perfect two terminal components and connecting leads or connections are used, which is far from the truth in practice.

Wien Capacitance Bridge. This bridge is shown in Fig. 15. Two adjacent arms of this bridge have nonreactive resistors, and the other two arms have capacitors respectively in series and in parallel with resistors. It is used to determine capacitance in terms of resistance and frequency. The balance equation is

$$R_1\left(\frac{R_x}{1+j\omega C_x R_x}\right) = R_2\left(R_3 + \frac{1}{j\omega C_3}\right) \tag{61}$$

By solving this equation, we obtain

$$C_x = \frac{R_1 C_3}{R_2(1+\omega^2 R_3^2 C_3^2)} \quad \text{and} \quad R_x = \frac{R_2(1+\omega^2 R_3^2 C_3^2)}{\omega^2 C_3^2 R_1 R_3} \tag{62}$$

This Wien bridge is also widely used in electronics, for example, to realize low distortion RC oscillators. In this case, the balance conditions may be rewritten

$$\frac{C_x}{C_3} = \frac{R_1}{R_2} - \frac{R_3}{R_x} \quad \text{and} \quad \omega^2 = \frac{1}{C_3 C_x R_3 R_x} \tag{63}$$

Maxwell-Wien Bridge. This bridge is normally used for the measurement of inductance in terms of resistance and capacitance. It was originally introduced by Maxwell as a ballistic method and at first used as an ac bridge by Wien (3,4). This bridge is shown in Fig. 16. The balance equation is

$$\frac{R_1(R_x + j\omega L_x)}{(1+j\omega C_1 R_1)} = R_2 R_3 \tag{64}$$

or if we separate the real and imaginary parts:

$$L_x = C_1 R_2 R_3 \quad \text{and} \quad R_x = \frac{R_2 R_3}{R_1} \tag{65}$$

It can be deduced that the balance is independent of frequency.

Hay Bridge. This bridge is particularly suitable to measure inductances with large time-constants. It differs from the Maxwell-Wien bridge in that in the arm opposite to the inductor, the capacitor is in series with the resistor. The circuit arrangement is given in Fig. 17. At balance,

$$\left(R_1 + \frac{1}{j\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

Solving for L_x and R_x,

$$L_x = \frac{R_2 R_3 C_1}{(1+\omega^2 R_1^2 C_1^2)} \quad \text{and} \quad R_x = \frac{\omega^2 R_1 R_2 R_3 C_1^2}{(1+\omega^2 R_1^2 C_1^2)} \tag{66}$$

These balance conditions are frequency dependent. The frequency must be well known and very stable. This is obtained nowadays very easily by using frequency synthesizers.

Schering Bridge. Schering first suggested this bridge in 1920. This bridge is used for the measurement of capacitance and dissipation factor. The unknown capacitor C_x and the standard loss-free capacitor C_2 form two adjacent arms, while the opposite arm to the capacitor C_x consists of a resistor R_1

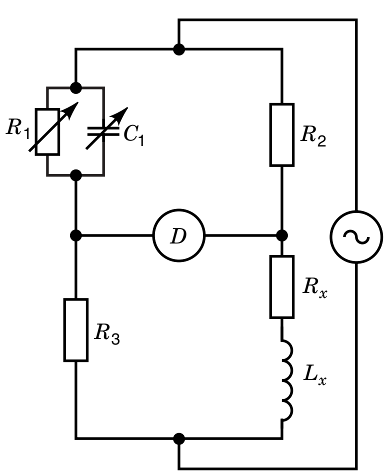

Figure 16. A Maxwell-Wien bridge.

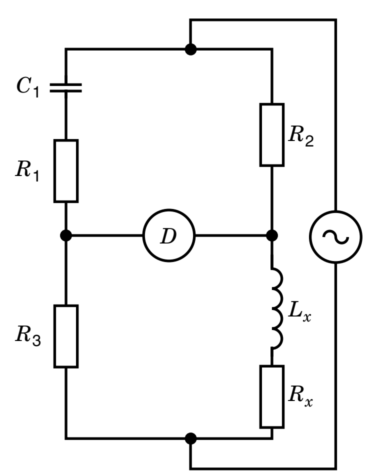

Figure 17. A Hay bridge.

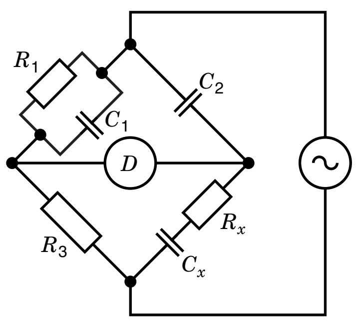

Figure 18. A Schering bridge.

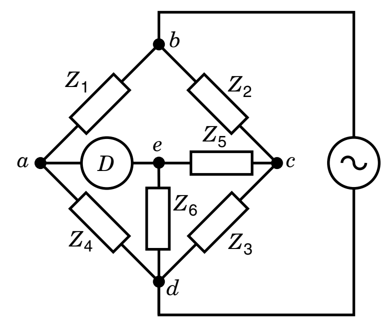

Figure 20. An Anderson bridge.

and a capacitor C_1 in parallel. The fourth arm is a nonreactive resistor R_3. The balance may be written referring to Fig. 18:

$$\left(R_x + \frac{1}{j\omega C_x}\right)\left(\frac{R_1}{1 + j\omega C_1 R_1}\right) = \frac{R_2}{j\omega C_2} \tag{67}$$

By separating the real and the imaginary parts:

$$C_x = \frac{C_2 R_1}{R_2} \quad \text{and} \quad R_x = \frac{C_1 R_2}{C_2} \tag{68}$$

The balance conditions are independent of frequency.

Wagner Principle. Already in many dc bridges but still more in ac bridges, the problem of leakage currents, coming, for example, from the source directly to the detector due to the poor isolation from the conductor of the mains supply called ground or earth or due to stray capacitances, must be under control. In many ac bridges, this is achieved by adding an auxiliary circuit or branch called Wagner earth to the bridge. It can be deduced from Fig. 19 that the admittances of the bridge and the auxiliary circuit respect the following conditions:

$$\frac{Y_1}{Y_4} = \frac{Y_2}{Y_3} = \frac{(Y_5 + Y_b)}{(Y_6 + Y_d)} \tag{69}$$

then the detector terminals are at the same potential as the earth or ground potential without being connected to the earth or to the ground. The admittances Y_a, Y_c and Y_b, Y_d have, consequently, no effect because they are respectively connected between equipotential points and in parallel with the admittance Y_5 and Y_6 of the auxiliary arms. The condition (69) is fulfilled by balancing successively the bridge constituted by Y_1, Y_2, Y_3 and Y_4 and the bridge consisting of Y_1, Y_4, Y_5 and Y_6 or of Y_2, Y_3, Y_5 and Y_6. The procedure is iterated until a consistent balance is obtained for the two possible connections of the detector.

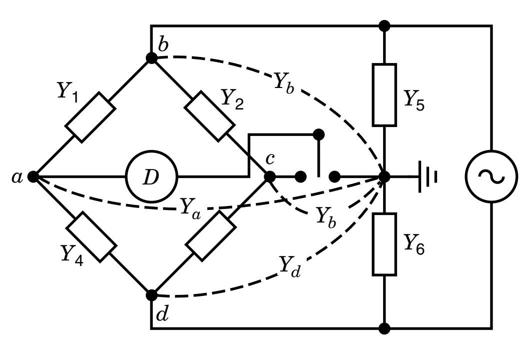

Figure 19. The Wagner principle.

Multiple-Arm Bridges

Other ac bridges having six arms and even more rarely seven arms have been described in the technical literature and also used successfully (3). However, it will be shown that several of these networks may be reduced to the basic four-arm configuration by suitable $Y - \Delta$ or $\Delta - Y$ transformations.

Anderson-Type Bridge. The interest of the modification introduced by Anderson has been demonstrated in the Maxwell-Wien bridge to measure inductance in terms of capacitance and resistance. If we apply the theorem of Kennelly to the triangle *cde* of the network shown in Fig. 20, we have the following equations:

$$Z_3' = \frac{Z_5 Z_6}{Z_3 + Z_5 + Z_6} \tag{70}$$

$$Z_5' = \frac{Z_3 Z_6}{Z_3 + Z_5 + Z_6} \tag{71}$$

$$Z_6' = \frac{Z_3 Z_5}{Z_3 + Z_5 + Z_6} \tag{72}$$

The general balance equation of the network represented in Fig. 21 is

$$Z_2 Z_4 (Z_3 + Z_5 + Z_6) + Z_3 Z_5 Z_4 = Z_1 Z_3 Z_6 \tag{73}$$

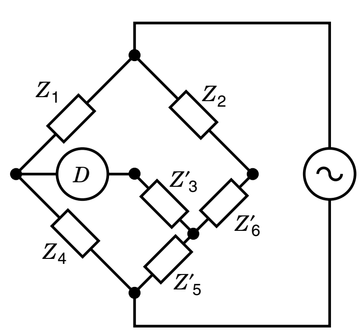

Figure 21. A modified Anderson's type bridge after a $Y - \Delta$ transformation.

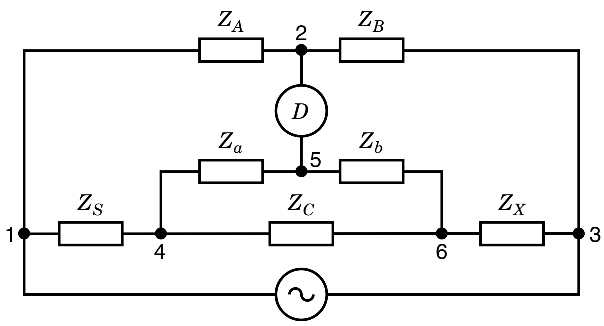

Figure 22. An ac Kelvin double bridge.

Ac Kelvin Double Bridge. The need to characterize four terminal low value resistors and, in particular, to know their time-constants and their residual inductances has led to extend the use of the Kelvin double bridge from dc to low frequency ac. A general form of this bridge is given in Fig. 22. Let us apply again the theorem of Kennelly to the network 456:

$$Z'_a = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \tag{74}$$

$$Z'_b = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} \tag{75}$$

$$Z'_c = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \tag{76}$$

The general equation of the network shown in Fig. 23 is

$$(Z_a + Z_b + Z_c)(Z_A Z_X - Z_B Z_S) = Z_c(Z_B Z_a - Z_A Z_b) \tag{77}$$

Therefore, the double condition as follows permits the achievement of balance:

$$\frac{Z_S}{Z_X} = \frac{Z_A}{Z_B} \quad \text{and} \quad \frac{Z_A}{Z_B} = \frac{Z_a}{Z_b} \tag{78}$$

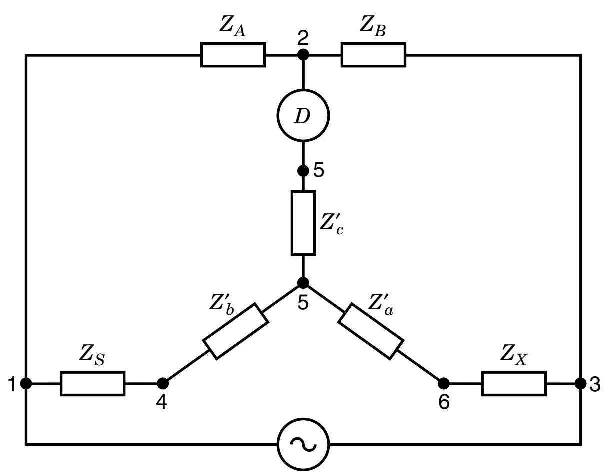

Figure 23. A modified ac Kelvin double bridge after a $Y - \Delta$ transformation.

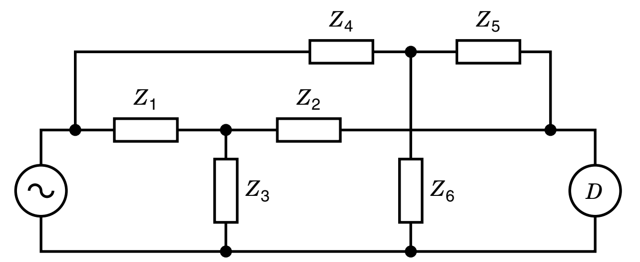

Figure 24. A Twin-T network.

Twin-T Networks. Networks other than the four-arm bridges have been developed to compare impedances. An important one among them is the Twin-T network. This circuit illustrated in Fig. 24 may be regarded as a parallel-T network. The direct impedances Z'_6 and Z'_3 between nodes 1 and 2 can be calculated after $Y - \Delta$ transformations (Fig. 25). If these direct impedances Z'_6 and Z'_3 are equal in magnitude and have phase angles differing by π radians, the currents through these impedances cancel one another at the input of the detector. The other admittances obtained after these transformations do not affect the balance condition because they shunt either the source or the detector. The balance condition of this parallel-T network is given by the following equation:

$$Z_1 + Z_2 + Z_1 Z_2 Z_3 + Y_4 + Z_5 + Z_4 Z_5 Y_6 = 0 \tag{79}$$

Therefore the null at the detector is only possible if the impedances contain reactances. This type of bridge is particularly suitable for use at high frequencies. An interesting feature of this type of network is the fact that the source and the detector have a common point. Therefore, the bridge balance is not disturbed by stray capacitances.

Transformer-Ratio Arm Bridges

In the four-arm or the multi-arm ac bridges, which have been examined before, the ratio arms are generally realized by fixed nonreactive resistances in a definite numerical ratio. For forty years, ac bridges have been designed where the impedance ratios in adjacent arms are advantageously replaced by voltage ratios from inductive voltage dividers (IVDs). An IVD is an accurate autotransformer provided with a variable tapping point.

At present, 8- or 9-decade IVDs with a departure from the nominal ratio of the number of turns of the windings lower than one part in ten million in the audio-frequency range are commercially available. Some of them are also correctly op-

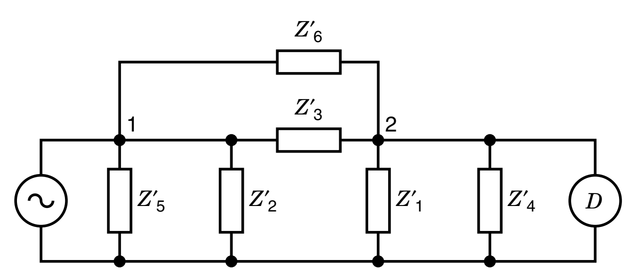

Figure 25. $Y - \Delta$ transformations applied to a Twin-T network.

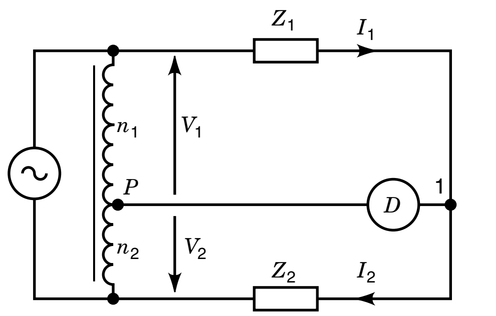

Figure 26. A potential transformer-ratio arm bridge.

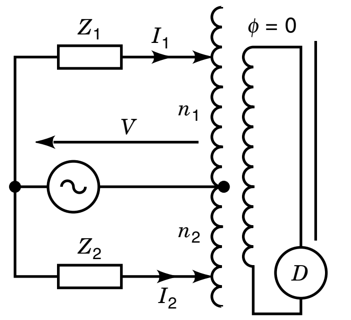

Figure 27. A current transformer-ratio bridge.

erating up to 1 MHz. Applications using 30 bits binary voltage dividers have also been described (20). The automatization of the IVDs seems also to be a promising feature for these devices.

Ratios defined by IVDs are very stable as a function of time, temperature, humidity, and even frequency. At 1 kHz, uncertainties lower than 3 parts in 10^8 for the ratio and less than 1 μrad for the phase are currently achieved for IVDs manufactured by qualified specialists in the field.

In transformers especially designed for ratio measurements, the departure from the ideal transformer has mainly the following origins:

- Power losses due to the magnetization and eddy currents in the core
- Power losses due to Joule effect, skin effect, and eddy currents in the windings
- Flux leakages
- Leakage inductances and capacitances

Potential Transformer-Ratio Arm Bridge. A simple example of a bridge, which might be called potential transformer bridge, is shown in Fig. 26. The balancing condition is easily obtained. Indeed, at node 1,

$$I_1 = I_2 \tag{80}$$

$$\frac{V_1}{Z_1} = \frac{V_2}{Z_2} \tag{81}$$

If the potential transformer is ideal,

$$\frac{V_1}{V_2} = \frac{n_1}{n_2} \tag{82}$$

$$\frac{Z_1}{Z_2} = \frac{n_1}{n_2} \tag{83}$$

In the potential transformer-ratio bridge, the ratio of the number of turns of the windings n_1 and n_2 determines the ratio of the voltage across the other arms. It can also be observed from the same diagram that, if the variable tapping point P is grounded, the stray capacitances associated with Z_1 and Z_2 are in parallel with the internal impedances of the two parts of the potential transformer. As these impedances are in general rather low, the admittances of these stray capacitances do not affect the balance of this simple bridge. If a lower uncertainty is requested, the generator must at least be isolated; a Wagner branch and an injection network have to be added to correctly balance this bridge.

Current Transformer-Ratio Bridge. This bridge, represented in Fig. 27, is the dual bridge of the potential transformer-ratio bridge. This bridge is a three-winding type where the extra winding detects the magnetomotive force balance. The magnetic and electrostatic screens between the toroidal windings and the detector winding are not represented. These screens are often necessary to improve the accuracy of this type of bridge. Contrary to the potential transformer-ratio bridge, which operates with a finite magnetic flux in the magnetic core, the flux in the magnetic core of a current transformer ratio bridge is theoretically zero.

When the bridge is balanced, the magnetomotive forces created by the currents flowing through the two impedances and the two windings nullify each other. At balance, we have

$$n_1 I_1 - n_2 I_2 = 0 \tag{84}$$

or

$$\frac{n_1}{Z_1} = \frac{n_2}{Z_2} \tag{85}$$

Ac Current Comparator Bridge

Many applications of the ac current comparator have been described for low as well as for high voltage and current applications (10). In low voltage applications, an ac current comparator is used in the development of many useful instruments like current transformers, active capacitor/quadrature current references, transconductance amplifiers, resistance-ratio ac bridges (21), and power comparators. We will describe this last one, which is also known as power bridge due to its use for the measurement of power and energy. The basic circuit, shown in Fig. 28, offers the possibility to provide a power standard (22). This bridge compares the magnetomotive forces created by the current I in the winding N_x with the magnetomotive forces in the winding N_r due to the in-phase current flowing through the resistance R and that in the winding N_c due to the quadrature current through the capacitor C. The balance condition is

$$IN_x = \frac{V}{R} N_r \pm j\omega C V N_C \tag{86}$$

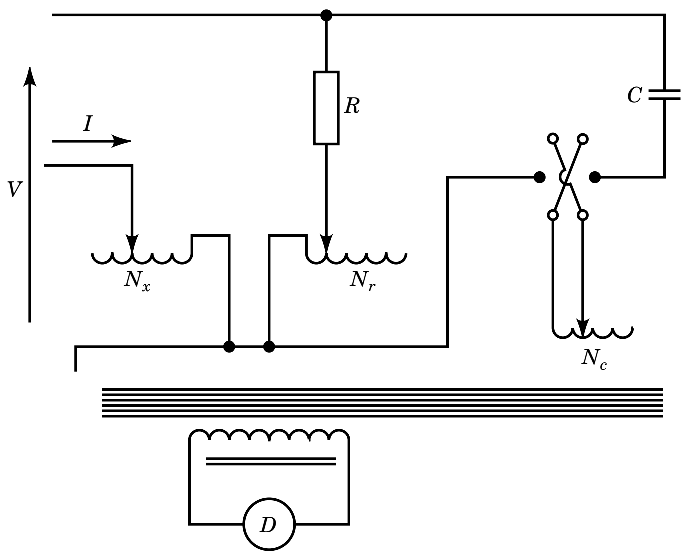

Figure 28. An ac current comparator bridge.

By rewriting Eq. (86),

$$VI = \frac{1}{N_x}\left(\frac{V^2}{R}N_r \pm j\omega CV^2 N_c\right) \tag{87}$$

This bridge measures the apparent power VI that has one active component proportional to V^2/R and one reactive component proportional to ωCV^2.

Coaxial ac Bridges

This is the latest refinement in the field of ac bridges. Measurement uncertainties as low as 1 part in 10^9 for impedance comparison can be achieved from about 200 Hz to 2 kHz with these kinds of coaxial ac bridges. However, the technique can be extended from 10 Hz to slightly above 1 MHz. The technique of the coaxial ac bridges is based on some of the following features:

1. Complete equality in magnitude and opposition in sign between the currents flowing in the inner and the outer conductor of the coaxial cable. By this means, no external field is created, and no electromagnetic interference is influencing the bridge.
2. Low impedance of the outer conductor of the cable and of the screens of the measuring devices
3. Fully screened or shielded measuring devices provided with terminal-pairs

Coaxial ac bridges are used to realize the link between the farad and the ohm and vice versa. A simple example of a coaxial ac bridge is shown in Fig. 29, with the permission of one of the authors (18). This bridge built around a calibrated IVD can be used to compare two terminal-pair measuring devices of similar admittance with a measurement uncertainty lower than 1 part in 10^6.

Automatic RLC Bridges

Many of the ac bridges described in the previous chapters are not very widespread in industrial laboratories and not very practical for component manufacturers. In many sectors of activities, the preference is mainly given to automatic *RLC* bridges, which are more easily available due to the fact that several manufacturers commercialize them. These *RLC* bridges offer several advantages:

- Automatic and quick measurement of the real and imaginary parts of the impedance vector and conversion of them into the desired parameters like $|Z|$, $|Y|$, Θ, R, X, G and B
- Computer control by means of an IEEE 488 interface
- Frequency range from a few hertz up to several megahertz

A schematic diagram of an HP 4284A Precision *LCR* Meter is represented in Fig. 30 with the permission of the manufacturer (23). As indicated in this figure, the structure of this bridge allows the measurement of 4TP devices. Coaxial cables are used to isolate the voltage sensing cables from the signal current path. Since the same current flows, but in an opposite direction through the outer conductor of the same coaxial cable, the magnetic field generated by the inner conductor is canceled. The outer conductor of the coaxial cable is also connected at the system ground.

This bridge uses the auto balancing method, where the current flowing through the device under test is balanced by the range resistor current in order to maintain a zero potential at the low terminal of the device under test. A null detector in amplitude and in phase measures the zero potential at this point. If the zero potential is not achieved, the null detector adjusts the amplitude and/or the phase of the second oscillator in order to maintain this balance condition. This bridge,

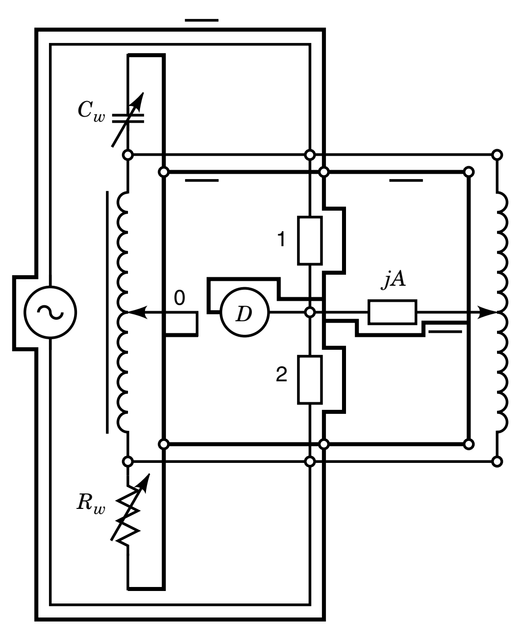

Figure 29. A two terminal-pair bridge based on IVDs [From (18) with the consent of Dr. B. P. Kibble and the permission of IOP].

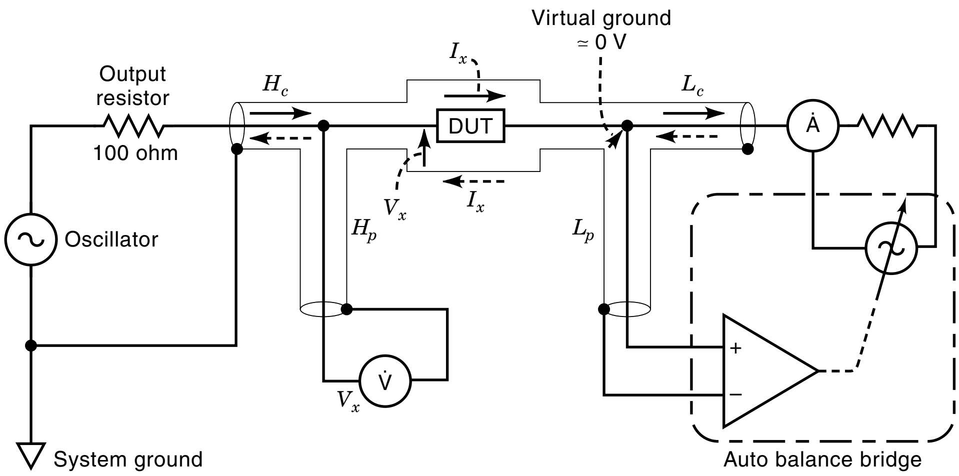

Figure 30. A schematic diagram of HP 4284A Precision *LCR* Meter (copyright Hewlett-Packard).

operating up to 1 MHz, allows the measurement of devices with a basic accuracy of 0.05% at 1 kHz. Better performances could be achieved when using a substitution method and standards calibrated with lower uncertainties. Before using this type of bridge, it is also needed to proceed to an OPEN/SHORT/LOAD compensation technique. The length of the cables must be taken into account, especially if the frequency is above the kilohertz band.

Ac Sources

From the balance conditions, it can be deduced that the first two requirements for an ac bridge source are the frequency stability and an output voltage sine waveform with very low distortion, especially if the balance condition is frequency dependent. The output power delivered by the ac bridge source may be insufficient, and the need for an additional power amplifier possibly associated with a matching transformer may perhaps be necessary. The addition of these devices will increase the total harmonic distortion of the source.

Modern *RC* audio-frequency sinewave generators usually present output voltages with a total harmonic distortion lower than 0.01% and generally offer the possibility to be phase-locked by a frequency synthesizer. Up to several kilohertz, as long as the total harmonic distortion of the output can be kept lower than 0.1%, it is also possible to work directly with a frequency synthesizer. Indeed, several of the commercially available frequency synthesizers in the audio frequency range have a frequency resolution higher than 1 part in 10^{10}. If necessary, the reference signal generated by the internal crystal oscillator may be replaced by the one produced by a higher stability oscillator like an atomic frequency standard. Digital signal generators may also be found useful due to their frequency and amplitude stability. However, the possible presence of glitches at the output of some of them may be a source of problems in the balancing process (24).

Ac Detectors

Passive detectors like the telephone receiver and the vibration galvanometer, which were very common at the beginning of ac bridge measurements, are no longer used. At present, nearly all detectors used in ac bridge measurements are fed from the mains supply or more rarely from batteries. The most important technical requirements for ac detectors are

- Amplitude sensitivity in current or in voltage
- Frequency selectivity and the ability to reject harmonics
- Phase sensitivity

It is to be noted that in ac, the experimenter can often use a matching amplifier between the network and the available detector.

The most common types of detectors are described below.

Wideband Detectors. Wideband detectors like oscilloscopes and ac millivoltmeters can be used. The wideband operation of these instruments can lead to the necessity of reducing the bandwidth by placing a band-pass filter or tuned preamplifier at the input of these detectors in order to achieve a better selectivity or signal-to-noise ratio.

Narrowband Detectors. Frequency-selective detectors, which are tuned to respond to the fundamental frequency of the in-phase and quadrature components of the out-of-balance signal, are also helpful. Their deflection will correspond to the sum-of-square of these components. Some commercially available detectors of this type also have the advantage of being battery operated. Nevertheless, their principle of operation can make the balancing of some bridges with poor convergence more difficult.

Phase-Sensitive Detectors. This type of detector is also known as a synchronous or lock-in detector. A phase-sensitive detector requires an auxiliary reference signal taken directly from the bridge source. It uses this signal to operate an electronic switch or a modulator. The most suitable detector of this type multiplies the incoming signal with the reference signal. The resulting signal is then passed through an integrating low-pass filter. Dual phase-sensitive detectors incorporated in the same instrument are also very convenient: one detector can be adjusted to respond to a signal with the same phase as the source, and the other to the quadrature component. In this way, the bridge convergence can be made more efficiently. This type of phase-sensitive detectors has the advantage of optimizing the recovery of the bridge out-of-balance signal from random noise. However, the direct connection from the source to the detector could be a source of problems if appropriate precautions, like the right use of an isolation transformer, are not taken.

Convergence of ac Bridges

The ability of a bridge to approach the balance is known as convergence. In ac bridges, this concept also includes the rate at which this balance is achieved. As seen previously, the balance condition is given by a system of two equations, which must be satisfied simultaneously. This indicates that in such bridges, at least two parameters, one reactive and one resistive, must be adjusted. The solution to this problem requires one to choose the adjusting devices and to determine the way to obtain the balance of the bridge in a minimum number of adjustments. As the best solution generally differs from one bridge to another, it is recommended to refer for this point to the bibliography (3,4).

Sensitivity of ac Bridges

As for the dc bridges we can define, at balance, the sensitivity of the detector as the ratio of the small deflection from zero resulting from a small change of the measured impedance to the value of the change. In this way, we can define the relative current sensitivity S_i as

$$S_i = \left| \frac{\Delta I}{\frac{\Delta Z}{Z}} \right|_{\Delta Z \to 0} \quad (88)$$

and the relative voltage sensitivity S_v as

$$S_v = \left| \frac{\Delta V}{\frac{\Delta Z}{Z}} \right|_{\Delta Z \to 0} \quad (89)$$

In theory, these values can be calculated from the parameters of the bridges including those of the detector (3,4). In practice, these parameters are easily measured by changing Z of a small amount from the balance and observing the small deflection ΔI or ΔV on the null detector.

Checking of ac Bridge Accuracy

Several principles explained for checking dc bridges could be applied for ac bridges if we take into account that we compare impedances and not pure resistors:

1. *Zero Voltage of the Bridge Source.* The same procedure as for dc bridges.
2. *Zero Impedance.* A suitable short circuit replaces also the device under test.
3. *Infinite Impedance.* An open circuit replaces the device under test. In this case, care must be taken in order to prevent the flow of capacitive currents.
4. *Checks of the Bridge Linearity.*
   - *With Individual Resistors or Impedances.* Depending on the type of instrument, the substitution method or the transposition method can be used. These methods can be applied with all types of resistors or impedances. The used devices do not need to be calibrated. They must have only good short-term stability if the other influencing parameters can be kept constant.
   - *With Resistance or Capacitance Networks.* Linearity checks of low frequency ac resistance bridges can be estimated using a string of equal value resistors connected in series. At present, a resistance network close to Hamon's design offers a convenient solution for the calibration of the ac resistance bridges from dc to 10^4 rad/s with an accuracy of about 1 part in 10^8 for 100 Ω (25).

Linearity checks of capacitance bridges can be executed by a set of capacitors successively connected in series and in parallel. Usually, the calibration of IVDs and voltage transformers can be made using an auxiliary transformer or set of capacitors (18).

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