MICROMECHANICAL RESONATORS

RESONANT SENSORS

SILICON SENSORS

INTRODUCTION

Micromechanical resonators have been used for several years as a timebase in electronic and mechanical systems and as a sensing element for a wide range of applications such as pressure sensing and rotation (gyroscope). This article details the basic principles of resonant sensing and reviews the fundamentals involved in designing a micromechanical resonator. The various materials previously used to realize miniature resonant transducers are discussed after which the article concentrates on micromachined silicon resonators. As explained in the article, the fact single crystal silicon is not piezoelectric means alternative methods of exciting and detecting the resonator's vibrations must be fabricated as an integral part of the device. Various mechanisms can be used and these are reviewed in detail.

The quality factor of a micromechanical resonator is a figure of merit describing its resonance. The importance of the quality factor and the damping effects that limit it are discussed enabling the reader to gain an in depth knowledge of the design principles involved. Other important resonator behavioral characteristics such as non linear behavior are also included. The references present a survey of key resonant silicon sensors developed to date enabling the reader to access a full background to the technology.

MICROMECHANICAL RESONATORS

A resonator is a mechanical structure designed to vibrate at its resonant frequency. Micromechanical resonators are miniature structures with dimensions typically varying from submicron to a few hundred microns. The structure can be as simple as a cantilever beam, a fixed-fixed beam clamped at each end or a tuning fork. The frequency of the vibrations of the structure at resonance are extremely stable enabling the resonator to be used as a time base (the quartz tuning fork in watches for example), or as the sensing element of a resonant sensor. In each application the behavior of the resonator is of fundamental importance to the performance of the device.

A resonant sensor is designed such that the resonator's natural frequency is a function of the measurand (1, 2). The measurand typically alters the stiffness, mass or shape of the resonator hence causing a change in its resonant frequency. The other major components of a resonant sensor are the vibration drive and detection mechanisms. The drive mechanism excites the vibrations in the structure whilst the detection mechanism "picks up" these vibrations. The frequency of the detected vibration forms the output of the sensor and this signal is also fed back to the drive mechanism via an amplifier maintaining the structure at resonance over the entire measurand range. A typical resonant sensor is shown diagrammatically in Figure 1.

The most common coupling mechanism is for the resonator to be stressed in some manner by the action of the measurand. The applied stress effectively increases the stiffness of the structure which results in an increase in the resonator's natural frequency. This principle is commonly applied in force sensors (3), pressure transducers (4) and accelerometers (5). Coupling the measurand to the mass of the resonator can be achieved by surrounding the structure by a liquid or gas, or by contact with small solid masses. The presence of the surrounding media increases the effective inertia of the resonator and lowers its resonant frequency. Densitometers and level sensors are examples of mass coupled resonant sensors (1). One example of coupling to the resonator via the shape effect is a micromachined silicon pressure sensor which uses a hollow, square diaphragm supported at the midpoints of each edge as the resonator (6). The internal cavity within the diaphragm allows each side to be squeezed as applied pressure increases which alters the curvature of the diaphragm in turn altering its resonant frequency.

MICROMECHANICAL RESONATOR MATERIALS

Since the design and behavior of the resonator is a major influence on the performance of the sensor, the properties of the material from which it is constructed are of fundamental importance. This section discusses the suitability of some common materials used to fabricate micromechanical resonators: silicon, polysilicon and quartz and gallium arsenide. The material properties of these materials are given in Table 1.

Single crystal silicon

Silicon is a single crystal material possessing a face centered diamond cubic structure. Silicon atoms are covalently bonded with 4 atoms bonded together forming a tetrahedron. As these tetrahedra combine they form a large cube or unit cell as shown in Figure 2. It is an anisotropic material with many of its physical properties, such as Young's Modulus, varying with the crystalline direction. Directions and planes within silicon are identified using the Miller Indices. The indices use the principle axes within the crystal and the form of bracket to denote each feature. For example (111) denotes a particular plane that bisects the x, y and z axes at 1, as shown in Figure 2.

An important feature of silicon is that it remains elastic up to fracture exhibiting no plastic behavior and therefore no hysteresis. This is an ideal material characteristic for resonant structures, since any plastic deformation would permanently alter the shape, and therefore resonant frequency, of the structure. Single crystal silicon is also intrinsically very strong. Given the fact it is elastic to fracture, the practical strength of silicon is however extremely dependent upon the number and size of crystalline defects, and the quality of the surface. Given silicon wafers with a low level of defects, sensible structural design, careful han-

J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*.

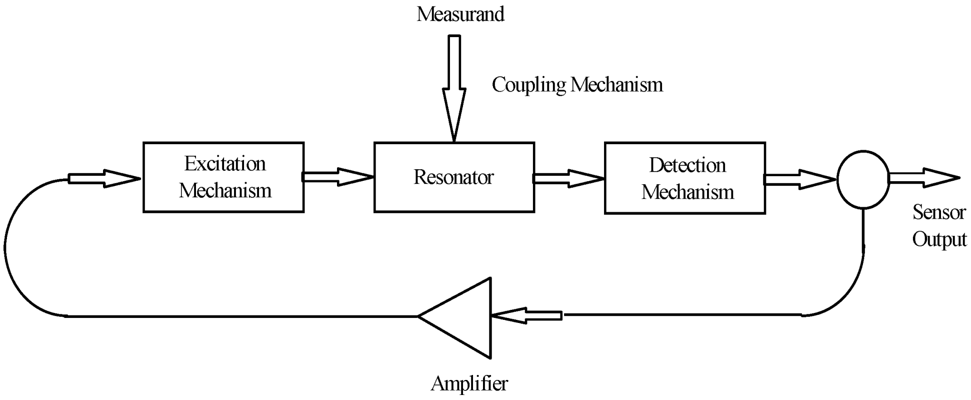

Figure 1. Block diagram of resonant sensor layout

Table 1. Material properties for common resonator materials

	Young's Modulus (N/m^2)	Fracture Strength (N/m^2)	Density (kg/m^3)	Thermal Expansion ($10^{-6}/°C$)	Thermal Conductivity (W/cm °C)
Silicon	1.9×10^{11}	7×10^9	2330	2.33	1.57
Quartz[1]	9.7×10^{10}	8.4×10^9	2650	0.55	0.014
Stainless Steel	2×10^{11}	2.1×10^9	7900	17.3	0.329
GaAs	8.53×10^{10}	2.7×10^9	5360	6.4	0.44

[a] Values given for Z cut quartz

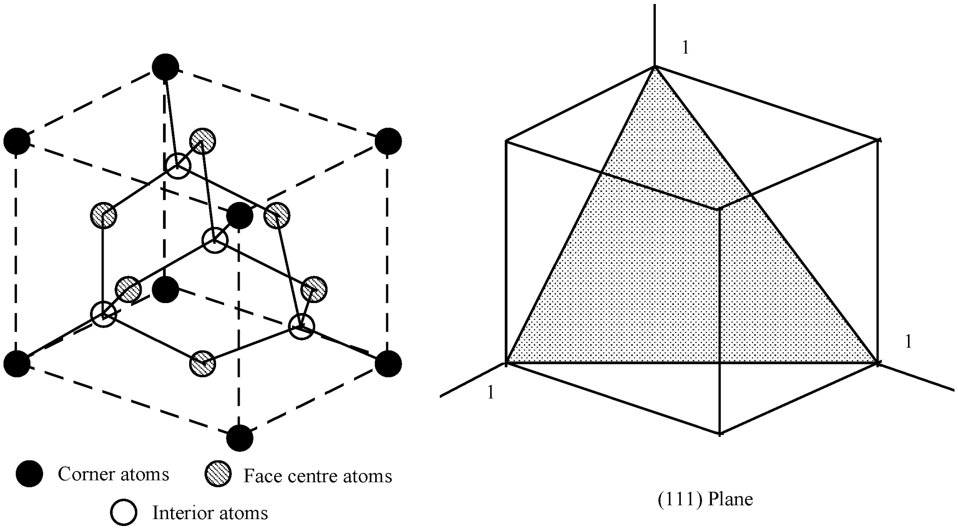

Figure 2. Diamond cubic unit cell and the (111) plane

dling and processing, strengths close to intrinsic and far in excess of alloyed steels can be obtained. The mechanical fatigue characteristics of silicon are also excellent. Fatigue can result in structures fracturing at applied stresses much lower than should otherwise cause such failure. Fatigue failure begins at a microscopic level with a minute defect or crack in the material which propagates due to fluctuating applied stresses. The varying stresses caused by the flexural vibrations of a resonating structure would be a typical environment for such fatigue to occur.

Other important favorable considerations include the low temperature cross sensitivity of resonant frequency, being measured at -29ppm/°C (7). The long term stability of silicon resonators have also been tested in numerous applications, and these results show no significant frequency shift over long periods of time (8). Silicon wafers are readily available in a variety of sizes, currently from 50 mm to 300 mm in diameter. Wafers are relatively cheap in their basic form, especially considering that hundreds of devices can be realized on each wafer. Standard wafers are avail-

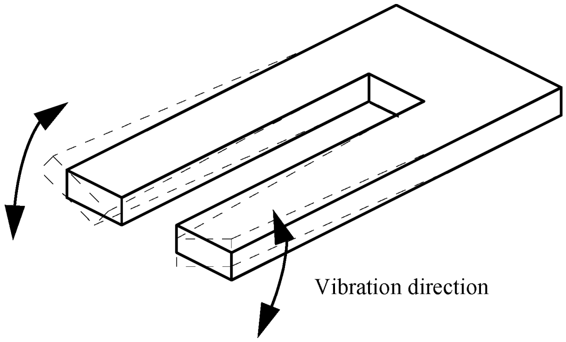

Figure 3. Quartz tuning fork resonator vibrating in a torsional mode

able in two forms distinguished by the crystal plane from which the wafer is cut, the most common wafer orientation being (100). More expensive wafers are available that include buried layers of silicon dioxide and these are useful for the fabrication of many types of micromachined devices, including resonant sensors.

A drawback associated with silicon with regard to its application as a resonator material is that it is not piezoelectric. When as voltage is applied to a piezoelectric material it deforms, and when it is forced to deform a potential gradient is generated. Piezoelectric behavior is ideal for exciting and detecting vibrations. In order to excite and detect vibrations in silicon resonators, alternative mechanisms such as electrostatic forces or thermal effects have to be employed. Whilst their use is well established, it does however complicate the fabrication of the resonator in comparison to similar structures fabricated from a piezoelectric material such as quartz.

Polysilicon

Polysilicon is an alternative material suitable for resonator fabrication and, as with single crystal silicon, much of the process development for polysilicon has been carried out by the semiconductor industry. It is often used where electronics are being integrated on the sensor chip due to the compatibility of the fabrication processes. In many applications, however, the material properties required for polysilicon micromechanical devices are different to those required in the fabrication of microelectronic devices. This is especially true in the case of resonant sensing where the mechanical properties of the polysilicon will play a vital role in sensor performance. Also reproducibility and repeatability of these properties is essential.

Polysilicon is a film of silicon atoms deposited upon the top surface of a substrate typically using chemical vapor deposition (CVD) processes. The film either has a granular or amorphous structure depending upon the process parameter. In the case of resonator applications, the film is typically deposited on a sacrificial layer of silicon dioxide. The polysilicon layer is then patterned and the sacrificial layer removed to leave the freestanding resonant structure. This is called surface micromachining.

The deposition process is of prime importance in determining the resulting mechanical properties of the deposited film. Process parameters such as substrate temperature, gas flow rate, deposition rate, deposition pressure and reactor design all affect the structure and behavior of the film. Another key factor affecting the performance of a resonator fabricated from polysilicon is the amount of residual stress built into in the layer during deposition. Residual stresses result from thermal expansion coefficient mismatches between the film and substrate, and also from the grain growth process which can trap atoms in positions that induce stress in the lattice. For resonant structures such stresses will naturally alter the performance of the device and will make prior modeling inaccurate. Residual stresses can be controlled and reduced by annealing the deposited films.

The mechanical properties of polysilicon will vary depending upon the process parameters, but general values have been reported for Poisson's ratio, Young's Modulus and tensile strength of 0.226, $1.75 \pm 0.21 \times 10^{11}$ Nm^{-2} and 1×10^{9} Nm^{-2} respectively (9). The long term stability of the material is reported to be good, with resonators fabricated in polysilicon being tested over 7000 hours of temperature cycling showing no detectable change in resonant frequency. Also, no signs of fatigue failure have been reported. Such polysilicon films are well suited to resonant sensor design as long as residual stresses are controlled. The repeatability of the film's properties will also be an important consideration when attempting to mass produce a resonant silicon sensor. Like single crystal silicon, polysilicon is not piezoelectric and vibration excitation and detection mechanisms must be fabricated at wafer level.

Quartz

The application of quartz as a resonant sensor material has evolved from its use as a crystal control for oscillating circuits. Quartz crystals were first used in radio communications equipment and now can be widely found in many applications, most noticeably as the timebase in clocks and watches (10). An important quality of quartz is its piezoelectric behavior. Piezoelectric materials deform when an electric field is applied and conversely generate a potential field when forced to deform. Quartz is a piezoelectric material because it possesses groups of atoms with an unbalanced charge, these being known as dipoles. These dipoles are permanently orientated in the same direction and quartz is therefore permanently polarized. When external stress is applied to the material its crystal structure is deformed. This deformation shifts the dipoles changing the polarization of the material and inducing a voltage in the process. Conversely, if an external voltage is applied to the crystal the orientation of the dipoles is changed deforming the crystal structure. Silicon is not piezoelectric because it possesses a covalently bonded symmetrical structure and therefore has no dipoles.

Quartz resonators employ the piezoelectric effect to excite the vibrations of the resonant structure. An applied alternating electric field will produce strain in alternating directions and this is a very simple way of exciting vibrations. A quartz resonator consists of a precisely dimensioned resonant structure with electrodes patterned on its surface. The nature of the vibration excited by the applied field depends upon the geometry of the resonator, its crystalline orientation and the electrode pattern (11). Resonators can be designed so that torsional vibrations can also be simply excited. Torsional vibrations are advantageous since they suffer less damping effects within the material.

Quartz possesses many other attractive features. It has excellent stability and long term aging characteristics. It possesses excellent material properties and resonators can be to batch fabricated using photolithographic techniques similar to those used in silicon. It also benefits from very low temperature cross sensitivity in certain crystal orientations.

The main drawbacks of quartz when compared to silicon revolve around its fabrication technology and the fact integrated circuitry cannot be formed on the sensor chip. Also quartz wafers are more expensive than silicon wafers. Nevertheless, quartz in an excellent resonator material and a wide range of quartz resonant sensors exist. Applications include sensing temperature, thin film thickness, force, pressure and fluid density.

Gallium Arsenide

Gallium arsenide (GaAs) is used in the semiconductor industry for high frequency, high temperature (>125 °C) electronic and opto-electronic applications. As with silicon, the development of the material for these applications has benefited the microengineering community. It is especially well suited to resonant sensing applications since it is piezoelectric and electronic circuitry can also be integrated onto the sensor chip. It may therefore be considered to combine the benefits of quartz and silicon as a resonator material.

GaAs atoms are arranged in a zincblende crystal structure and are held in place by ionic bonds. The presence of the arsenide atoms in the lattice draws the electrons and this results in dipoles orientated along the <111> directions (12). These dipoles result in the piezoelectric nature of GaAs. As with quartz, longitudinal, flexural, torsional and shear vibrations can be excited piezoelectrically and this makes it well suited for resonant applications.

GaAs is a brittle material and, as with silicon, its mechanical strength will ultimately depend the presence of stress concentrating defects. GaAs wafers contain more crystalline defects than single crystal silicon wafers, and structures will therefore not be as strong as their silicon counterparts. Tests have been carried out on GaAs cantilever beams which failed at 2.7 GNm^{-2} (13), whilst this is well below the values obtained for silicon structures it is nevertheless stronger than steel.

There are a wide range of micromachining techniques available including various wet and dry etches, bonding, and selective etch stops. Structures can also be fabricated on GaAs substrates using thin films of aluminum gallium arsenide (AlGaAs) and sacrificial layers as described in the case of polysilicon structures (14). Internal stresses mainly due to thermal mismatches are however a problem with this technique. There are certainly many more micromachining options available for GaAs resonant sensors than for their quartz counterparts.

The main drawbacks associated with GaAs as a microresonator material result from the increased costs involved in its use. GaAs wafers are more expensive than silicon wafers and processing costs are also higher than for the equivalent silicon operation. Silicon also possesses a much wider range of processes. Silicon and Quartz are also well characterized materials and their use as resonator materials is long-standing and fully documented. The use of gallium arsenide in MEMS applications is still relatively uncommon.

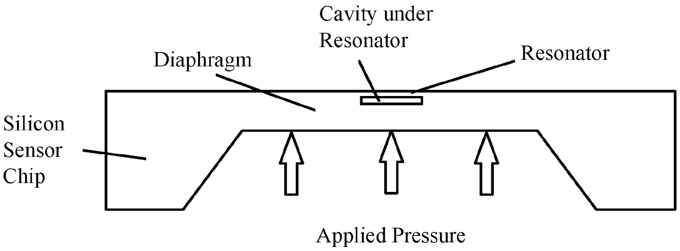

Figure 4. Resonant pressure sensor configuration

RESONANT SILICON SENSORS

The mechanical properties of the pure silicon wafers used in the fabrication of integrated circuits and the ability to form microstructures using some of the processes developed by the semiconductor industry, has enabled silicon to be used in a variety of microengineered devices. At the forefront of such microengineered devices are silicon sensors, including resonant silicon sensors.

In the case of micromachined silicon resonant sensors, the resonator can be fabricated from silicon and the vibration drive and detection mechanisms fabricated on, or adjacent to, the resonator. The resonator is then located in a silicon structure specifically designed for the sensing application. This is illustrated by the approach used in the design of the majority of resonant silicon pressure sensors where the resonator is fabricated on the top surface of a silicon diaphragm (see figure 4). In such a case the resonator is being used as a strain gauge sensing the strain induced due to the deformation of the diaphragm caused by an applied pressure (15).

The use of silicon overcomes the major disadvantage of traditional resonant sensors, namely the manual fabrication of individual resonating elements. Fabricating the sensor in silicon enables the resonator to be batch fabricated and hundreds of devices can be realized on each silicon wafer simultaneously. The wafers themselves can also be processed simultaneously in the majority of the fabrication steps.

COMPARISON OF SILICON SENSING TECHNOLOGIES

Silicon strain sensors monitoring deflections in the sensor structure can employ piezoresistive, capacitive or resonant sensing techniques. Of these, resonant sensing is inherently the more complex approach. This is highlighted by the fact the vibration excitation and detection mechanisms are commonly based upon piezoresistive or capacitive techniques. However, the typical performance figures for the three strain sensing techniques listed in

Table 2. Performance features of resonant, piezoresistive and capacitive sensing

Feature	Resonant	Piezoresistive	Capacitive
Output Form	Frequency	Voltage	Voltage
Resolution	1 part in 10^8	1 part in 10^5	1 part in 10^4–10^5
Accuracy	100–1000 ppm	500–10000 ppm	100–10000 ppm
Power consumption	0.1–10 mW	≈ 10 mW	< 0.1 mW
Temperature cross sensitivity	$-30 \times 10^{-6}/°C$	$-1600 \times 10^{-6}/°C$	$4 \times 10^{-6}/°C$

table 2 clearly show the advantages resonant sensing offers.

VIBRATION EXCITATION AND DETECTION MECHANISMS

There are many potential vibration excitation and detection mechanisms possible in silicon. Many of the mechanisms listed below can be used to both excite and detect a resonator's vibrations, either simultaneously or in conjunction with another mechanism. Devices where a single element combines the excitation and detection of the vibrations in the structure are termed one-port resonators. Devices which use separate elements are termed two-port resonators.

The choice of mechanism for driving or detecting a resonator's vibrations depends upon several important factors. These factors include the magnitude of the drive forces generated, the coupling factor (or drive efficiency), sensitivity of the detection mechanism, the effects of the chosen mechanism upon the performance and behavior of the resonator and practical considerations pertaining to the fabrication of the resonator and the sensor's final environment.

Electrostatic excitation and detection

The electrostatic excitation of a resonator relies upon electrostatic forces between two plates. These plates may be separated either by an air gap with one plate being located upon the resonator and the other located on the surrounding structure. Alternatively a three layer sandwich can be formed on resonator's surface consisting of two electrodes separated by a dielectric layer. The associated vibration detection mechanism relies upon the change of capacitance between the electrodes as the resonator deflects. One drawback to be considered when using the dielectric sandwich on the resonator surface is the increase in the temperature cross-sensitivity of the resonator.

Lateral vibrations in the plane of the wafer have also been excited using the electrostatic mechanism combined with a comb structure (16). The resonator must be designed with suitable comb fingers that align with comb fingers fabricated adjacent to the resonator. The resonator is typically fabricated from polysilicon on the top surface of the wafer.

One port resonators using the same pair of electrodes to both excite and detect vibrations (17), or two port configurations using separate electrode pairs (18) have both been achieved. Electrostatic excitation is also commonly used alongside alternative vibration detection techniques e.g. piezoresistive (19).

Piezoelectric excitation and detection

This approach uses a deposited layer of piezoelectric material to both excite and detect the resonator's vibrations (20). The piezoelectric material, typically zinc oxide (ZnO), is formed in a sandwich between two electrodes creating a piezoelectric bimorph. Applying an oscillating voltage to the bimorph causes periodic deformation thereby exciting the resonator. Detection is provided by the corresponding potential generated by the deformation in the bimorph. The piezoelectric bimorph is formed and patterned on the resonator surface in a manner similar to the dielectric electrostatic mechanism.

Magnetic excitation and detection

This technique places a current carrying resonator in a permanent magnetic field, the excitation being due to Lorenz forces acting upon the resonator. Applying an alternating current to the resonator results in alternating forces and hence vibrations are induced in the resonator. The associated detection mechanism utilizes the change in magnetic flux caused by the resonator moving in the magnetic field (21).

Electrothermal excitation

This approach relies upon the heating effect caused by passing a current through a patterned resistor located on the surface of the resonator. The heat energy generated causes a thermal gradient across the resonator's thickness, with the top surface at a higher temperature than the bottom. The induced thermal expansion of the material results in a bending moment on the resonator thereby deforming the structure. Vibrations can simply be excited by modulating the current through the resistor. Electrothermal excitation is widely used and can be found on many resonant devices.

Optothermal excitation

The thermal drive technique described above can also achieved using the heating effect resulting from a light source incident on the resonator. Modulating the light source will result in periodic thermal expansion on the surface of the resonator and hence induce vibrations in the structure (22). The light source is commonly aligned over the resonator using an optical fiber and this method can be conveniently used in conjunction with optical vibration detection techniques.

Optical methods can also be used to obtain self oscillation of the resonator using unmodulated light (23). The unmodulated light source is directed on to the resonator via an optical fiber, the end of which forms a Fabry Perot interferometer with the reflective surface of the silicon resonator. The interferometer output varies as a function of the resonator's displacement and this has the effect of modulating the incident light as the resonator vibrates. It is however difficult to sustain these vibrations and it requires critical and stable alignment of the fiber.

Optical excitation techniques are attractive since they enable a passive resonant structure with no on chip excitation mechanisms required. The use of optical fibres is compatible with miniature nature of micromachined silicon devices they are immune to electromagnetic interference can allow resonant silicon sensors to be used in harsh, high temperature environments. However the accurate integration and alignment of the optical fiber onto the sensor chip is difficult to achieve, especially in mass produced sensors.

Optical detection

There are several optical detection techniques suitable for monitoring the vibrations of a resonator. This approach was developed using optical fibre to couple the light to the resonator which enabled it to be readily combined with the optical drive mechanisms. More recently, advances in on-chip waveguides offer the potential for incorporating optical detection mechanisms at wafer level, albeit light still has to be coupled in from an external source in the devices demonstrated (24). Interferometric techniques rely upon two interfering reflections, one from a fixed reference and the other from the vibrating resonator. The combination of the reflected light results in interference fringes the number of which gives a direct measure of the amplitude of displacement. Intensity modulation techniques use changes in the intensity of the reflected light to monitor the amplitude of the resonator vibrations. The reflections can be modulated for example by the angular displacement of the resonator as it vibrates.

Piezoresistive Detection

This uses the inherent piezoresistive nature of silicon to detect the vibrations of the resonator. Resistors can be fabricated by diffusing or implanting them into the silicon, or by depositing polysilicon resistors on the top surface of the resonator. The resistor can simply be connected in a Wheatstone bridge circuit, the value of the resistor varying as the resonator vibrates. The frequency of the changing resistance forms the output of the sensor and hence the actual value of the resistance and the behavior of the resistor becomes largely unimportant. This approach is simple to achieve being widely used in many resonant sensors (25), is compatible with integrated circuit fabrication techniques, and can be readily used either with electrothermal excitation or an alternative drive excitation mechanism such as electrostatic.

THEORY OF RESONANCE

In order to understand resonance the propagation of mechanical waves within a solid must be understood. A mechanical wave may be defined as the propagation of a physical quantity (e.g. energy or strain) through a medium (solid or fluid) without the net movement of the medium. As the wave travels through the medium particles are displaced from their equilibrium position thereby distorting the medium. The form of the wave will depend upon the nature of its source and the material through which it travels. The speed of a wave in a solid is dependent upon the mechanical properties of the material, and its wavelength is a function of the frequency of the wave source.

As the wave travels through a solid it can meet boundaries or regions of non-uniformity of material or geometrical form. Upon meeting such discontinuities the nature of the wave will be changed due to phenomena known as refraction, diffraction, reflection and scattering. When considering the phenomena of natural frequencies and resonance, reflection of a mechanical wave at a system boundary becomes important. Reflection is the reversal in direction of the wave back along its original path.

If the reflected wave exactly coincides with the incoming wave then a standing wave is created. The superposition of these waves results in the amplitude of each wave combining to become twice that of the initial wave. Taking a string fixed at each end as an example, a mechanical wave will be reflected back from the fixed boundary at each end. The standing wave phenomena will occur at specific frequencies known as the natural, or modal, frequencies of the fixed-fixed string. For each natural frequency, the string will have a characteristic distorted form, or mode shape as shown in figure 5. These are the modes of vibration of the string and also the mode shapes associated with a fixed-fixed beam. Points of zero displacement are called nodes whilst the points of maximum displacement are known as antinodes. The amplitude of the standing waves will decline with time due to damping effects in the system. The amplitude will be maintained however if the wave source, for example a harmonic driving force, at that frequency is maintained. If a harmonic driving force is applied to a system and a mode of vibration is excited, then the system is in resonance. Resonance will occur when the frequency of the driving force matches the natural frequency of the system. Resonance is a phenomena associated with forced vibrations whilst natural frequencies are associated with free vibrations. The motion of the oscillating body will not usually be in phase with the driving force. When resonance has occurred the motion of the oscillating body will lag the driving force by a phase angle of $\pi/2$.

QUALITY FACTOR

As a structure approaches resonance the amplitude of its vibration will increase, its resonant frequency being defined as the point of maximum amplitude. The magnitude of this amplitude will ultimately be limited by the damping effects acting on the system. The level of damping present in a system can be defined by its Quality Factor (Q-factor). The Q-factor is a ratio of the total energy stored in the system to the energy lost per cycle due to the damping effects present:

$$\mathrm{Q} = 2\pi(\text{maximum stored energy}/\text{energy lost per cycle}) \quad (1)$$

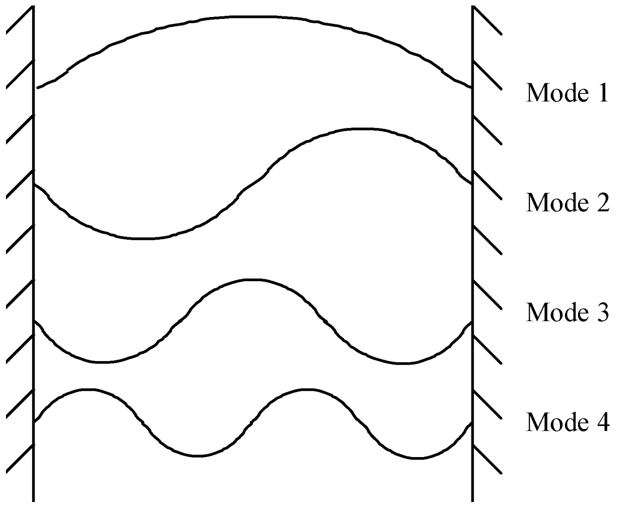

Figure 5. First 4 modes of vibration of a fixed-fixed string

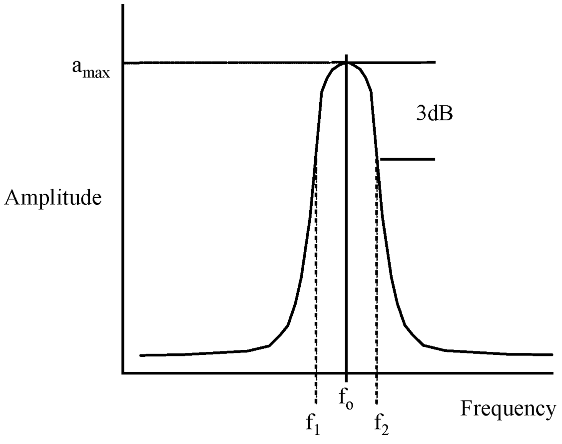

Figure 6. Typical amplitude frequency relationship

A high Q factor indicates a pronounced resonance easily distinguishable from non resonant vibrations, as illustrated in Figure 6. Increasing the sharpness of the resonance enables the resonant frequency to be more clearly defined and will improve the performance and resolution of the resonator. It will also simplify the operating electronics since the magnitude of the signal from the vibration detection mechanism will be greater than that of a low Q system. A high Q means little energy is required to maintain the resonance at constant amplitude thereby broadening the range of possible drive mechanisms to include weaker techniques. A high Q factor also implies the resonant structure is well isolated from its surroundings and therefore the influence of external factors, e.g. vibrations, will be minimized.

The Q-factor can also be calculated from:

$$Q = \frac{f_0}{\Delta f} \tag{2}$$

Where resonant frequency $f_o$corresponds with a_{max}, the maximum amplitude, and Δf is the difference between frequencies f_1 and f_2. Frequencies $f_1$and f_2 correspond to amplitudes of vibration 3dB lower than a_{max}.

From equation (1) it is clear the Q-factor is limited by the various mechanisms by which energy is lost from the resonator. These damping mechanisms arise from a number of sources identified in the equation below:

$$Q = (\frac{1}{Q_m} + \frac{1}{Q_t} + \frac{1}{Q_c} + \frac{1}{Q_{su}} + \frac{1}{Q_f})^{-1} \tag{3}$$

where:

$1/Q_m$ is the dissipation arising from the material loss,
$1/Q_t$ is the dissipation arising from the thermoelastic loss,
$1/Q_c$ is the dissipation arising from the clamping loss,
$1/Q_{su}$ is the dissipation arising from the surface loss,
$1/Q_f$ is the dissipation arising from the surrounding fluid,
Minimizing these effects will maximize the Q-factor.

In addition:

$$\frac{1}{Q_f} = \frac{1}{Q_v} + \frac{1}{Q_a} + \frac{1}{Q_{sf}} \tag{4}$$

where:

$1/Q_v$ is the dissipation arising from the viscous or molecular region loss,
$1/Q_a$ is the dissipation arising from the acoustic radiation loss,
$1/Q_{sf}$is the dissipation arising from the squeeze film loss.

These various damping effect in resonators are now discussed in turn.

Material derived losses

Movement of dislocations and scattering by impurities. A dislocation is an imperfection within the crystal lattice and these fall into two categories, edge or screw dislocations. Attenuation of the resonator vibrations can occur due to movement of these dislocations. The magnitude of such losses depends upon the number of dislocations, the resonator's frequency and temperature. Only at higher temperatures (>150 °C) is there sufficient energy present to move the dislocations. Also silicon is available in a very pure form and typical wafers contain a very low number of dislocations. Given likely operating temperature ranges and the low dislocation density this form of internal damping can effectively be ignored in silicon resonators.

Impurities are foreign atoms either inadvertently trapped in the lattice during crystal growth or intentionally added during the fabrication process. The presence of these impurities can result in point defects within the lattice introducing regions of inelastic scattering of the structural wave within the solid and thus causing energy to be lost within the material. The levels of impurities present within silicon are low and they have been shown to have negligible effect below 200 °C (26). This effect can therefore also be ignored for typical device operating conditions.

Phonon interaction. Each atom contained within the crystal lattice vibrates, due to thermal energy, about a mean position. When determining the heat capacity and thermal conductivity of the solid, these atomic vibrations are viewed collectively as a series of traveling lattice waves known as phonons (27). These phonons are subject to the scattering effect of, and they can also be similarly scattered by, variations in the lattice strain field due to the presence of other phonons. This is known as phonon-phonon interaction. The mechanical wave forming the structure's resonance can hence interact with variations in the strain field due to the presence of phonons. This results in the acoustic energy being converted to thermal energy, i.e. phonons are created.

Akheiser loss (28, 29). At lower frequencies Akheiser theory must be used to analyze the damping effects of phonons upon the mechanical wave. The thermal phonons within the material are analyzed as a gas described by a number of macroscopic parameters. The energy loss mechanism arises from the influence of the structural wave of the resonator on the phonon gas. The structural wave of the resonator will introduce fluctuations in the strain field of the material causing a modulation of the phonons. The modulated phonons collide with each other, and with impurities, and this has the effect of throwing the phonon gas out of equilibrium. This is an irreversible process and therefore energy is lost from the resonator's vibrations.

Thermoelastic effect

Flexural vibrations of a resonator lead to cyclic stressing of its top and bottom surfaces. The majority of the energy employed in displacing the beam is stored elasticity but some of the work is also converted into thermal energy. Material on the surface in compression will rise in temperature whilst the surface in tension falls in temperature. Hence a temperature gradient is formed across the thickness of the beam. If the deformation is maintained for a sufficient length of time, heat energy will flow across the gradient in order to equilibrate the system. Any thermal energy transferred in this manner is lost to entropy and the resonator's vibrations are attenuated.

The magnitude of this effect is dependent upon the mechanical properties of the resonator material, temperature and the resonant frequency. The damping fraction (δ=1/2Q) can be expressed as a product of two different functions, Γ(T) and Ω(F) (30):

$$\delta = \Gamma(T)\Omega(F) \tag{5}$$

$$\Gamma(T) = \frac{a^2 TE}{4\rho C} \tag{6}$$

$$\Omega(F) = 2[\frac{F_o F}{F_o^2 + F^2}] \tag{7}$$

where

α is the thermal expansion coefficient
T is beam temperature
E is Young's Modulus
ρ is material density

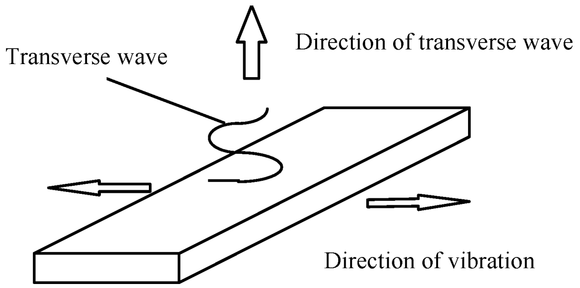

Figure 7. Formation of transverse wave

C is specific heat capacity
F is resonant frequency

F_o is the characteristic damping frequency given by:

$$F_o = \frac{\pi K}{2\rho C t^2} \tag{8}$$

where

t represents the beam thickness
K is the thermal conductivity of the resonator material

This form of damping can be reduced by designing a structure with a resonant frequency removed from F_o. Shear deformation, associated with torsional modes, does not suffer from thermoelastic damping.

Clamping friction or support loss ($1/Q_c$)

Clamping friction for the beam results from radiation of acoustic energy through the resonator's supports into the bulk of material surrounding the resonator. At resonance the vibrations set up a standing-wave pattern within the resonator. However, at the supports or ends of the resonator, some energy is lost outside the resonator decreasing the Q. This is caused by the shear force and moment on the clamped end. The shear force and moment act as vibration sources for launching elastic waves into the support. To avoid this, the motion of the supports should be minimised. This is achieved for tuning forks or double ended tuning forks in some particular balanced vibrational modes, in which the two beams vibrate in antiphase. Finite element analyses (31) have provided the optimum DETF geometry to minimise this source of losses. An analytical model has been derived for the support loss in micromachined beam resonators for clamped-free and clamped-clamped geometries (32) for in plane flexural vibrations.

The coupling mechanism between the resonator and its support can be illustrated by observing a fixed-fixed beam vibrating in its fundamental mode. Following Newton's second law, every action has an equal and opposite reaction, the reaction to the beam's vibrations is provided by its supports. The reaction causes the supports to deflect and as a result energy is lost from the resonator. This is shown in Figure 8.

The degree of coupling of a fixed-fixed beam can be reduced by operating it in a higher order mode. For example the second mode in the plane of vibrations shown above

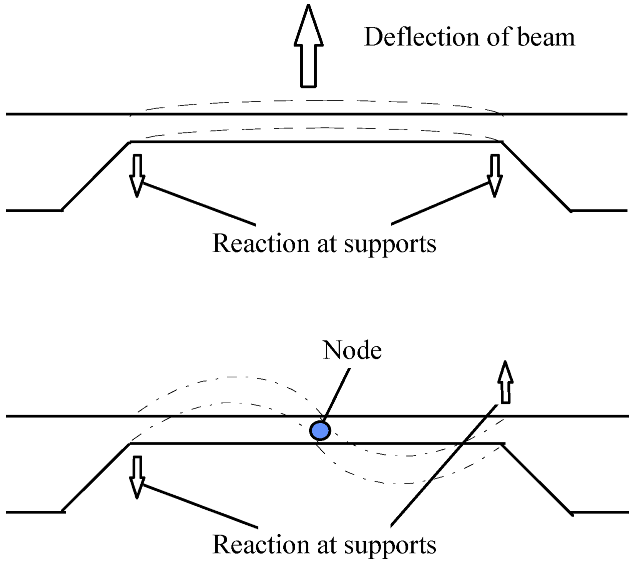

Figure 8. Support reactions a) Fundamental mode b) Second mode

will possess a node half way along the length of the beam. The beam will vibrate in antiphase either side of the node and the reactions from each half of the beam will cancel out at the node. There will inevitably still be a reaction at each support, but the magnitude of each reaction will be less than for mode 1. The use of such higher order modes is limited by their reduced sensitivity to applied stresses and the fact there will always be a certain degree of coupling.

Balanced resonator designs operate on the principle of providing the reaction to the structure's vibrations within the resonator. Multiple beam style resonators for example incorporate this inherent dynamic moment cancellation when operated in the balanced mode. Examples of such structures are the Double Ended Tuning Fork (DETF) which consists of two beams aligned alongside each other and the Triple Beam Tuning Fork (TBTF) which consists of three beams aligned alongside each other, the center tine being twice the width of the outer tines. Figure 9 shows these structures and their optimum modes of operation.

$1/Q_C$is of fundamental importance since it not only affects the Q-factor of the resonator, but provides a key determinant of resonator performance. A dynamically balanced resonator design that minimizes $1/Qc$ provides many benefits:

- High resonator Q-factor and therefore good resolution of frequency.
- A high degree of immunity to environmental vibrations.
- Immunity to interference from surrounding structure resonances.
- Improved long term performance since the influence of the surrounding structure on the resonator is minimized.

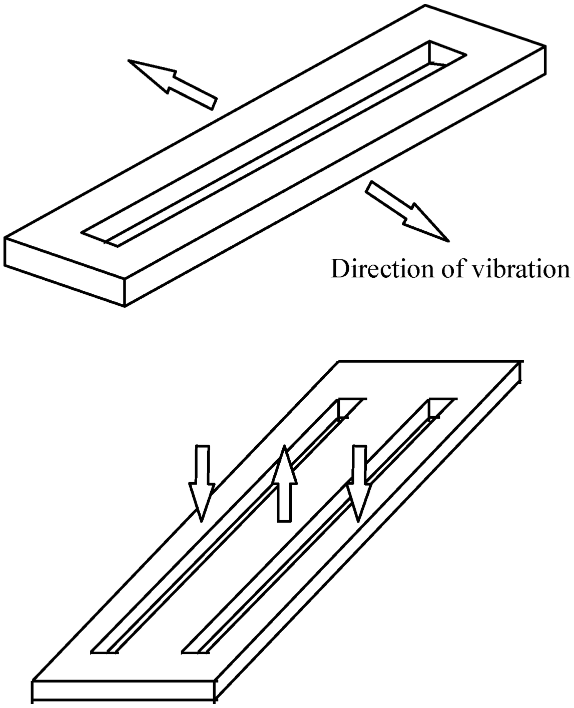

Figure 9. DETF and TBTF resonators

Surface loss ($1/Q_{su}$)

Surface loss is particularly relevant to nanoscale devices since as the surface to volume ratio increases the surface loss become more significant (33). The surface loss is mostly caused by surface stress which can be significantly enhanced by absorbates or surface defects.

Air damping resulting from viscous drag or molecular collisions ($1/Q_v$)

A resonator vibrating in a viscous fluid produces movement of the fluid, with fluid velocity components both parallel and perpendicular to its surface and propagating perpendicular to its surface. Both components give rise to loss. The parallel component causes the loss due to viscous drag and produced a frictional force in phase with the resonator velocity (34).

An early paper (35) associated the viscous drag with two pressure regions. In region 1, the lower pressure region, the fluid molecules are so far apart they do not interact with each other. The individual air molecules exchange momentum with the resonator at a rate proportional to the difference in velocity between the molecules and the resonator. In the second pressure region the fluid molecules interact like a viscous fluid.

The displacement of the fluid particles will also have an effect on the resonator's natural frequency. This is because the mass of the adjacent fluid particles is effectively added to the mass of the resonator and hence its resonant frequency will be reduced. The order of magnitude of the added mass effect is dependent upon the relative densities of the fluid and resonator material (36), and there is a much smaller dependence upon the fluid pressure and the aspect ratio of the structure (37).

Air damping resulting from acoustic radiation ($1/Q_a$)

Acoustic radiation occurs when the vibrations of the resonator causes small pressure variation in the fluid in contact with the resonator. These pressure variations effectively form a source of travelling acoustic waves with the same frequency as the vibrations of the resonator. The production of the perpendicular component, corresponding to acoustic radiation is efficient and results in a low Q when the wavelength of the acoustic waves in the resonator is approximately equal to the acoustic wavelength in the fluid at the frequency of vibration. It has been shown (38) that the acoustic radiation loss is zero if the wavelength of flexural waves within the resonator is less than the wavelength of acoustic waves within the fluid.

Squeeze film damping ($1/Q_{sf}$)

Squeeze film damping occurs between two normally oscillating plates separated by a small gap (39). Assuming a fluid within the gap the pressure has two components one in phase with the drive to the oscillating plate and another in phase with the velocity of the oscillating plate. These respectively represent the spring like behaviour of the gas and the damping of the gas.

This form of damping often occurs in electrostatically driven resonators where close positioning of the drive and detection electrodes is essential. In such an arrangement it can become the major damping mechanism (40).

The magnitude of this effect depends upon the frequency of operation, the pressure of the surrounding fluid and the geometry of the resonator. The frequency dependence arises from the time taken to displace the fluid. At low frequencies the fluid has time to move between the surfaces and energy is lost from the resonator's vibrations. As the frequency rises however, some of the fluid at the center of the resonator does not have enough time to move and becomes trapped between the surfaces. As the frequency rises further an increasing proportion of the fluid remains trapped. The trapped volume of fluid acts as a spring resulting in a slight increase in resonant frequency. As the frequency continues to rise the spring effect becomes increasingly predominant and the damping effect progressively declines (41). The squeeze film effect can be alleviated to a certain extent by incorporating apertures in the resonator allowing some fluid transport through the resonator.

NON LINEAR BEHAVIOR

Non linear behavior becomes apparent at higher vibration amplitudes when the resonator's restoring force becomes a non-linear function of its displacement. This effect is present in all resonant structures. In the case of a flexurally vibrating fixed-fixed beam the transverse deflection results in stretching of its neutral axis (42). A tensile force is effectively applied and the resonant frequency increases. This is known as the hard spring effect. The magnitude of this effect depends upon the boundary conditions of the beam. If the beam is not clamped firmly the non-linear relationship can exhibit the soft spring effect whereby the resonant frequency falls with increasing amplitude. The nature of the effect and its magnitude also depends upon the geometry of the resonator (43).

The equation of motion for an oscillating force applied to an undamped structure is given below:

$$m\ddot{y} + s(y) = F_o \cos\omega t \qquad (9)$$

where

m is the mass of the system,
F the applied driving force,
ω the frequency,
y the displacement
s(y) the non-linear function.

In many practical cases s(y) can be represented by:

$$s(y) = s_1 y + s_3 y^3 \qquad (10)$$

the non-linear relationship being represented by the cubic term.

Placing equation (10) in equation (9), dividing through by m, and simplifying gives:

$$\ddot{y} + s_1/m(y + s_3/s_1 y^3) = F_o \,\mathrm{Cos}\,\omega t \qquad (11)$$

Where s_1/m equals ${\omega_{or}}^2$ (ω_{or} representing the resonant frequency for small amplitudes of vibration) and s_3/s_1 is denoted β. The restoring force acting on the system is therefore represented by:

$$R = -\omega_{or}^2(y + \beta y^3) \qquad (12)$$

If β is equal to zero the restoring force is a linear function of displacement, if β is positive the system experiences the hard spring non-linearity whilst a negative β corresponds to the soft spring effect. The hard and soft non-linear effects are shown in Figure 10. As the amplitude of vibration increases and the non-linear effect becomes apparent, the resonant frequency exhibits a quadratic dependence upon the amplitude, as shown below

$$\omega r = \omega_{or}(1 + \frac{3}{8}\beta y_o^2) \qquad (13)$$

The variable β can be found by applying equation (13) to an experimental analysis of the resonant frequency and maximum amplitude for a range of drive levels.

The amplitude of vibration is dependent upon the energy supplied by the resonator's drive mechanism and the Q-factor of the resonator. Driving the resonator too hard or a high Q-factor that results in excessive amplitudes at minimum practical drive levels can result in undesirable non-linear behavior. Non-linearities are undesirable since it can adversely affect the accuracy of a resonant sensor. If a resonator is driven in a non-linear region then changes in amplitude, due for example to amplifier drift, will cause a shift in the resonant frequency indistinguishable from shifts due to the measurand. The analysis of a resonator's non-linear characteristics is therefore important when determining a suitable drive mechanism and its associated operating variables.

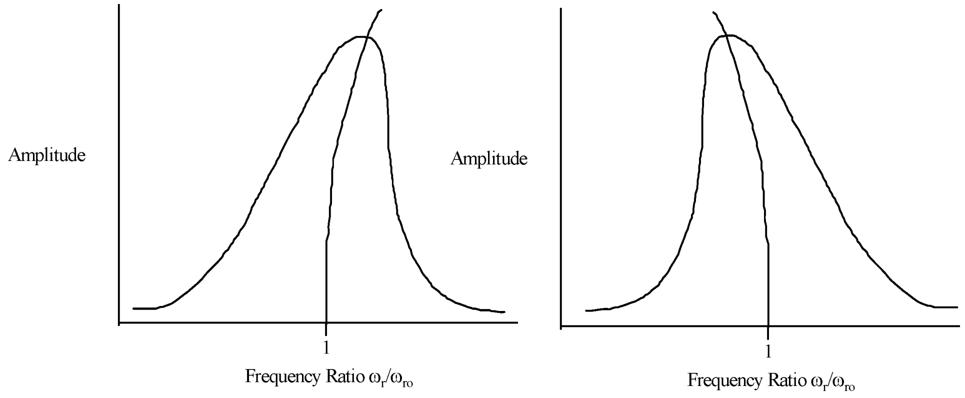

Figure 10. Hard and soft non-linearities

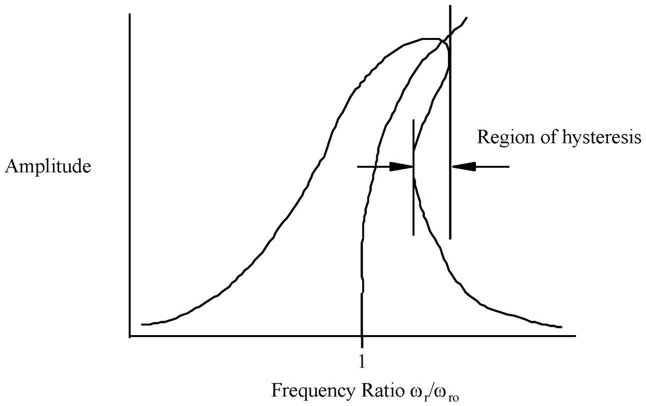

Figure 11. Frequency/amplitude relationship exhibiting hysteresis

HYSTERESIS

A non-linear system can exhibit hysteresis if the amplitude of vibration increases beyond a critical value. Hysteresis occurs when the amplitude has three possible values at a given frequency. This critical value can be determined by applying (43).

$$y_o^2 > \frac{8h}{3\omega_{or}|\beta|} \tag{14}$$

Where h is the damping coefficient and can be found by measuring the Q-factor of the resonator at small amplitudes and applying:

$$Q = \frac{\omega_{or}}{2h} \tag{15}$$

MICROMECHANICAL RESONANT SENSORS

A wide number of resonant microsensors have been demonstrated in the literature and some of these have been successfully commercialised. This section mentions just a few of the devices which have been reported. The main application of micromechanical resonators are in pressure sensors. These include the Druck resonant 'butterfly' structure positioned on a pressure sensing diaphragm (4) and the Yokogawa H-shaped resonator which employed a novel wafer level vacuum encapsulation process, again this resonator is mounted on a diaphragm and used to sense differential pressure (21). The exacting demands required by the oil and gas industry for down hole pressure sensing applications can only be met by resonant sensors. This market is currently dominated by a resonant quartz device supplied by Quartzdyne (44). A purely surface micromachined resonant beam pressure sensor has been demonstrated where the compact diaphragm and resonator design enables its use in catheter pressure sensing applications. The electrostatically operated resonant beam of dimensions 130 × 40 × 1.2 μm was coupled to a 150 × 100 × 2 μm diaphragm and the micromachining process includes on-chip vacuum encapsulation (45).

Vacuum sensing has been demonstrated using micromechanical resonators. The resonators are surrounded by the evacuated media and variations in the applied vacuum levels alter the damping effects which change the characteristics of the resonance (46, 47). The applied vacuum not only determines the Q-factor but also influences the resonant frequency of the device. Other applications include strain sensing employing a surface micromachined polysilicon DETF resonator (48) and a bulk micromachined resonant beam accelerometer (49) both of which operate atmospheric pressure and therefore have low Q-factors of 370 and 170 respectively.

MICROMECHANICAL RF RESONATORS

A more recent application of micromechanical resonators is as a timebase for radio frequency (1 – 1 GHz) applications. The advance of mechanical resonators into this field has been facilitated by the improvements in fabrication processes enabling ever smaller resonant structures, and therefore higher frequencies, to be realized. Micromechanical resonators are a potential replacement for quartz crystal oscillators which, despite their excellent performance, have the drawbacks of large size and incompatibility with on chip integration. To be successful in this application, the resonator should exhibit a high Q (tens of thousands), excellent stability across a range of temperatures and good signal to noise characteristics. The high Q values are required to meet phase noise standards for communications reference oscillators (e.g. 130 dBc/Hz at 1 kHz offset for a 10 MHz crystal oscillator reference) (50).

MEMS piezoelectric thin-film bulk acoustic resonators (FBAR's) consisting of thin films of piezoelectric material sandwiched between metal electrodes and operated in an extensional mode are already commercially successful (51). However, to achieve integration with electronics, nanoscale resonant structures have also been fabricated from polysilicon. These resonators are predominantly electrostatically excited into resonance and the vibrations detected capactively. This approach is non-thermal and non-contact (i.e. doesn't require dissimilar materials on the resonator structure) which are essential factors necessary to achieve the required levels of stability. A range of structures have been presented in the literature including a 10 MHz fixed-fixed beam (52), a wine glass disk structure resonating at 60 MHz (53), a contour mode disk resonator at 1.5 GHz (54) and a hollow disk structure (55) with a Q of over 15,000 at a frequency of 1.46 GHz.

CONCLUSIONS

Micromechanical sensors are miniature structures which can be used in both timebase and sensing applications. When used as a sensor, a variety of sensing principles can be employed to monitor a wide range of measurands such as pressure, acceleration, density and flow. Micromechanical resonator sensors use a micromachined resonator with dimensions in the range of tens of nanometres to hundreds of microns. The micromechanical resonator can be fabricated in batches from silicon, quartz and gallium arsenide wafers. Resonant sensors offer high accuracy, high resolution and high stability and, if designed correctly, should be superior to alternative sensing mechanisms e.g. piezoresistive, capacitive.

The frequency of the mechanical resonator typically changes with the measurand and the output of the sensor is thus a change in frequency. Mechanisms to excite the resonator's vibrations and detect them must be included in the sensor and a wide variety of techniques are available to the sensor designer. A simple method is the use of the piezoelectric effect in quartz.

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