J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*

PUMPED-STORAGE POWER STATIONS

General Description of Power System, Demand, and Reserve

A power system consists of power stations and transmission and distribution networks. The power stations in a utility can be classified as thermal, hydro, and pumped-storage stations, and each station usually has several generating units. A utility also purchases power from and sells power to other utilities. A utility must prudently schedule various units, and makes purchase and sale transactions to meet the demand in an economic and reliable way. Sales and purchases, though important, will not be discussed; and interested readers are referred to (1,2).

The sum of power required by all customers of a utility is *system demand*. Human activities have cycles, and so does system demand. In each day, there is usually a peak and an off-peak period as depicted in Fig. 1. The off-peak demand is much lower than that of the peak in a day (e.g., 50 to 60% of the peak demand). Within a week, the demand on weekends and holidays is generally lower than that of weekdays because of the reduced use of many factories and businesses that consume a significant portion of power.

A utility usually forecasts the demand, and then schedules its generating unit to satisfy it. It is possible, however, that a scheduled unit may unexpectedly break down, and starting another unit takes time. For the reliability of a system, some units have to carry a reserve, that is, they can increase the generation level, or be brought on-line in a matter of minutes. For some utilities, the reserve is required to be equivalent to the largest resource (unit, or purchase) on line, so that even if the largest resource becomes unavailable, the reserve capacity can replace the lost resource.

The cost to generate one megawatt-hour (MWhr) of power is quite different for different units. Hydro units are usually the most economical because no fuel is consumed for generation. For thermal units, steam units are generally more economical and efficient than gas turbines that usually have smaller capacities and are designed to burn the more expensive liquid or gaseous fuels (3). Also, new units are usually more fuel-efficient than old ones.

Starting a thermal unit incurs costs too, since a significant amount of fuel is required to warm up the unit. Starting a unit also causes wear and tear. These costs are lumped together as startup cost.

The goal of a utility is to satisfy the demand and reserve with minimum cost—the cost to generate and the startup cost. To satisfy the high demand at peak hours, some units, though uneconomical, may have to be generating at those hours. This leads to the well-known fact that the marginal cost (the cost for providing one more MWhr of power) at peak hours is much higher than that of the off-peak hours. Figure 2 shows the normalized marginal cost curve of a utility during a one-day period. It would be economical if a device can be designed such that it can store large quantity of cheap energy during off-peak periods, and then sets out the energy during peak periods when the energy is expensive. A pumped-storage power station, which will be the main topic in the remainder of this article, has this desirable characteristic.

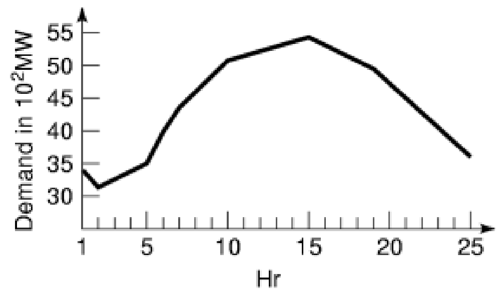

Fig. 1. Daily system demand with one peak and one off-peak.

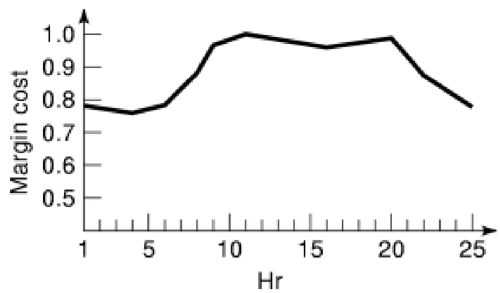

Fig. 2. Variation of the marginal cost (normalized).

The background knowledge of pumped-storage power stations will be presented in the first section of this article. Following this, a scheduling method focusing on pumped-storage power stations will be presented. Literature review and future research topics will then be addressed in the final section.

Characteristics of Pumped-Storage Power Stations

A pumped-storage power station has a lower reservoir, an upper reservoir, and a powerhouse with several generation units, as depicted in Fig. 3. The generation units in the powerhouse can operate in pumping mode to pump water from the lower reservoir to the upper reservoir during off-peak periods. Cheap energy is thus transferred into potential energy of the water at the upper reservoir. In peak periods, the units can operate in generation mode and then can discharge water from the upper reservoir to the lower reservoir through the turbine. Potential energy is thus transferred into electric power to satisfy the peak demand. Since some thermal units, such as nuclear and coal units, are very difficult to cycle, the capability of storing energy in off-peak periods is very important to system operation. Though pumped-storage stations cannot actually produce new energy, its economic benefit is significant.

Pumped-storage stations actually lose some energy due to its operation efficiency. If 1 MWhr of energy is used to pump a certain amount of water into the reservoir, the amount of energy available for generation is only ρ MWhr, where ρ is the efficiency coefficient (ρ is typically around 0.67 (3), and is 0.75 for Northfield station).

The net effect of pumped-storage stations is shaving the peak and filling the off-peak, and thus smoothing the demand curve. Besides, in generating mode, a unit can increase its generation level very quickly; in pumping, it can stop pumping and thus reduce the demand almost instantly. Pumped-storage units can therefore provide reserves to the system in both generating and pumping modes. For the Northfield station of Northeast Utilities, the reserve contribution as a function of the pumping/generating level is shown in Fig. 4. A pumped-storage unit can thus smooth the demand curve, provide reserve, and play an important role in reducing the total generating cost.

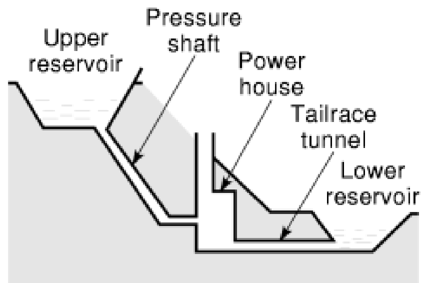

Fig. 3. A pumped-storage station consists mainly of reservoirs and a powerhouse with several generating units. In pumping state, water is pumped from the lower reservoir to the upper reservoir; and in the generating state, water is released from the upper reservoir to the lower reservoir through the powerhouse.

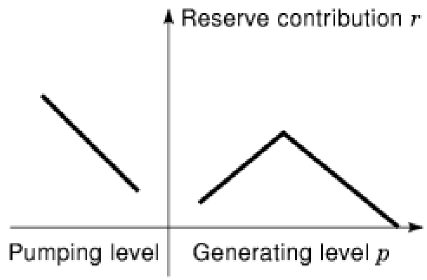

Fig. 4. The reserve contribution as a function of generation/pumping level for Northfield station. In pumping state, the unit can stop pumping to reduce demand, and thus provides reserve; in generating state, the unit can increase its generation level, and provide reserve.

Since the demand is cyclic, pumped-storage stations have an operation cycle. Generally, its operation cycle is one day or one week. The operation cycle of the Northfield station is currently one week. At the beginning of each cycle (8:00 a.m. on Monday), the upper reservoir level of the Northfield is full; and at the end of each cycle, its level is full again for the use of the next cycle.

The operation of a pumped-storage station is bounded by its physical laws and limitations. For the upper reservoir, its level at each hour should satisfy the following:

- The reservoir level at each hour should be greater than or equal to zero, but less or equal to the reservoir capacity. This is the reservoir level boundary constraint.
- Water balance equation should be observed, that is, volume change equals the amount of water discharged or pumped. This is the reservoir level dynamic constraint.

The lower reservoir is generally attached to a river with notable flow. For the Northfield station, the lower reservoir is connected to the Connecticut River. In the pumping mode, there is sufficient water in the lower reservoir for pumping, and in generating, the lower reservoir can discharge water to the river if it becomes full. It is thus not necessary to consider the constraints for the lower reservoir. Latter on, *reservoir* refers specifically to the upper reservoir.

Each unit in the powerhouse has its installed capacity. The generating or pumping level of the unit at each hour should be within the capacity. It is not efficient, or sometimes even prohibited, for a unit to generate or pump at very low level. The generating/pumping level should thus be within a lower and upper bound. A unit can also stay idle—neither generating nor pumping.

The several units in a pumped-storage station are usually integrated as an equivalent one in the modeling. The capacity of the equivalent unit is the sum of the capacity of individual units. Some utilities may have special requirements for the equivalent unit, such as that it can pump only at discrete levels.

Optimizing the Operation of Pumped-Storage Power Stations

The weekly operation cycle on an hourly basis will be used here. Optimizing the operation of a pumped-storage station involves deciding the hourly operation mode and generating/pumping level within a cycle. Pumped-storage power stations are components of a power system, and are operated in accordance with thermal and hydro units to satisfy the demand and reserve requirements at the minimum cost. An overall optimization is thus necessary.

The objective of the optimization is to minimize the total operation cost, which includes thermal fuel and startup cost. Usually, the operation cost of hydro and pumped-storage units is ignored in the optimization because of its insignificance. The optimization is subject to demand and reserve requirements, and individual unit constraints. The demand and reserve requirements are called system-wide constraints, because they couple all the units in the system together. The individual pumped-storage unit constraints were described in the previous section. Individual thermal and hydro unit constraints are described in Ref. 4.

The optimization objective and the constraints can be described in mathematical models, and then an appropriate optimization algorithm is developed (or used, if available) to solve the problem. The result of the optimization is a power system schedule that includes the mode and generating/pumping level at each hour for each pumped-storage unit, the on-off state and generating level at each hour for each thermal unit, and the generating level at each hour for each hydro unit.

The optimization problem belongs to the class of *NP*-hard (nonpolynomial) combinatorial problems, that is, the computational requirements increase exponentially with problem sizes. Consistently generating optimal schedules has proved to be extremely difficult for systems of practical sizes, however, a near-optimal scheduling can be obtained with Lagrangian relaxation technique (5). The mathematical model and an optimization algorithm based on Lagrangian relaxation will be presented in the next section.

Modeling and Optimization Methodology

Modeling.

Objective Function and System-Wide Constraints. Consider a power system with I thermal units, J hydro units, and k pumped-storage units. The objective is to minimize the total thermal operation cost, subject to system-wide demand and reserve requirements, and individual unit constraints.

The *objective function* to be minimized is the sum of thermal generation costs $C_{ti}(p_{ti}(t))$ and start-up costs $S_{ti}(t)$, that is,

$$\min_{p_{ti}(t), w_{hj}(t), w_{pk}(t)} J, \text{ with } J = \sum_{t=1}^{T} \left\{ \sum_{i=1}^{I} [C_{ti}(p_{ti}(t)) + S_{ti}(t)] \right\} \quad (1)$$

In the above, $p_{ti}(t)$ is the generation of thermal unit i at time t, $w_{hj}(t)$ the water released of hydro unit j, $w_{pk}(t)$ the water released of pumped-storage unit k (negative for pumping), and T is the time horizon (e.g., $T = 168$ hours for weekly scheduling). The function $C_{ti}(p_{ti}(t))$ is piece-wise linear.

The *system demand constraints* require that the sum of all thermal generation $p_{ti}(t)$, hydro generation $p_{hj}(w_{hj}(t))$, and pumped-storage generation $p_{pk}(w_{pk}(t))$ (negative for pumping) should equal the system demand

$p_d(t)$ at each hour, that is.,

$$\sum_{i=1}^{I} p_{ti}(t) + \sum_{j=1}^{J} P_{hj}(w_{hj}(t)) + \sum_{k=1}^{K} p_{pk}(w_{pk}(t)) = p_d(t), t = 1, \ldots, T \tag{2}$$

where $p_{hj}(w_{hj}(t))$ and $p_{pk}(w_{pk}(t))$ are water-power conversion functions for hydro unit j and pumped-storage unit k, respectively. The functional relationships are mostly assumed to be linear or quadratic in the literature for easy manipulating, and linear relationship is used here.

Reserve requirements state that the sum of reserve contributions of thermal units $r_{ti}(p_{ti}(t))$, hydro units $r_{hj}(p_{hj}(t))$,, and pumped-storage units $r_{pk}(p_{pk}(t))$ should be greater than or equal to the reserve required $p_r(t)$ at each hour, that is,

$$\sum_{i=1}^{I} r_{ti}(t) + \sum_{j=1}^{J} r_{hj}(w_{hj}(t)) + \sum_{k=1}^{K} r_{pk}(w_{pk}(t)) \geq p_r(t), t = 1, \ldots, T \tag{3}$$

Individual thermal unit constraints include capacity and minimum generation, minimum up/down times, ramp rate, and must-run and must-not-run. Individual hydro unit constraints include capacity and minimum generation, and available hydro energy. For detailed description of these constraints, please refer to Ref. 4.

Constraints for Pumped-Storage Units. The constraints include those related to the reservoir, and those related to the generating/pumping units. The physical significance of these constraints has been explained in an earlier section, and the mathematical modeling is presented here.

(1) Reservoir level dynamics: the level at hour $t + 1$ equal to the level at hour t minus the amount of water discharged (negative for pumped).

$$v_k(t+1) = v_k(t) - w_{pk}(t), t = 1, 2, \ldots T \tag{4}$$

where $v_k(t)$ is the reservoir level of pumped-storage unit k at time t, converted to MWhr.

(2) The reservoir level boundaries and the initial and terminal levels.

$$0 \leq v_{\mathrm{k}}(t) \leq \overline{V}_{\mathrm{k}}, t = 1, 2, \ldots T - 1 \tag{5}$$

where $\bar{V}_k$ is the maximum reservoir level of pumped-storage unit k.

$$v_k(0) = V_k^0 \tag{6}$$

$$v_k(T) = V_k^{\mathrm{T}} \tag{7}$$

where $V^0{}_{\mathrm{k}}$, $V^{\mathrm{T}}{}_k$ are initial and terminal levels, respectively, all converted to MWhr.

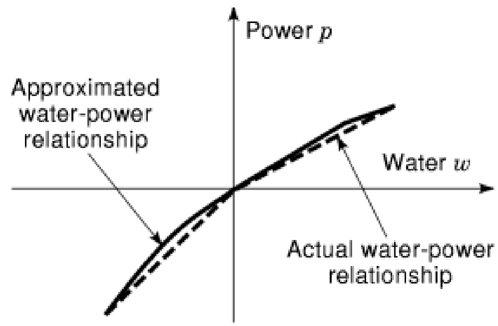

Fig. 5. Quadratic approximation of water/power relationship. Negative for pumping.

(3) Generating or pumping level constraints: The generating and pumping level constraints for unit k are given, respectively, by:

$$p^{g}_{-pk}(t) \leq p_{pk}(w_{pk}(t)) \leq p^{-g}_{pk}(t) \qquad (8)$$

$$-p^{-p}_{pk}(t) \leq p_{pk}(w_{pk}(t)) \leq -p^{p}_{-pk}(t) \qquad (9)$$

where $p^{g}{}_{-pk}(t)$ and $p^{-g}{}_{pk}(t)$ are generating boundaries at hour t, and $p^{p}{}_{-pk}(t)$ and $p^{-p}{}_{pk}(t)$ pumping boundaries. The unit can also be idle,

$$p_{pk}(w_{pk}(t)) = 0 \qquad (10)$$

Water-Power Conversion of Pumped-Storage Units. In the modeling, the water volume in the reservoir is measured by the energy needed to pump that amount of water. Suppose the efficiency coefficient of a pumped-storage unit is a constant ρ. The water power conversion function is then piece-wise linear as depicted in Fig. 5.

The Reserve Contribution of Pumped-Storage Units. The spinning reserve contribution of a pumped-storage unit is a piece-wise linear function of the generating/pumping levels as shown in Fig. 4 for Northeast Utilities, and it may be different for others. An important common fact is that pumped-storage stations can contribute significant reserve.

Optimization Methodology. The optimization for hydrothermal scheduling with pumped-storage unit is identified to have the following three major difficulties.

(1) It is a large-scale optimization problem. The system-wide constraints of Eqs. (2) and (3) couple the decision of each individual unit, and overall optimal scheduling cannot be obtained by simply optimizing each individual unit.
(2) The reservoir dynamic constraint of Eq. (4) couples the decision at each hour for the pumped-storage unit. The amount of water pumped/discharged will change the reservoir level at subsequent hours, and will then directly impact the decisions.
(3) It is a mixed integer-programming problem.

Lagrangian relaxation can overcome the difficulties. It is the system-wide constraints that couple the optimization of individual units, so the basic idea of Lagrangian relaxation is to relax the coupling system-wide constraints by multipliers. After relaxation, each individual unit can be optimized separately. The multipliers are updated at a high level to satisfy the once relaxed coupling constraints based on the solution of each individual unit.

In the method to be presented, the system-wide constraints will be relaxed and individual thermal, hydro, and pumped-storage subproblems will be constructed. In solving pumped-storage subproblems, the reservoir dynamic constraints which couple the optimization among hours will be relaxed by another set of multiplies. In solving the thermal subproblems, dynamic programming is used to handle integer decision variable (unit on or off) (4).

The Relaxed Problem. Relaxing system-wide demand and reserve requirements [Eqs. (2) and (3)] by using Lagrangian multipliers λ and μ, respectively, the following relaxed problem is formed:

$$
\begin{aligned}
\operatorname*{Min}_{p_{ti}(t), w_{hj}(t), w_{pk}(t)} & L, \text{ with } L \\
\equiv & \sum_{t=1}^{T} \left\{ \sum_{i=1}^{I} [C_{ti}(p_{ti}(t)) + S_i(t)] \right\} \\
& + \sum_{t=1}^{T} \lambda(t) \left[P_d(t) - \sum_{i=1}^{I} p_{ti}(t) \right. \\
& \left. - \sum_{j=1}^{J} p_{hj}(w_{hj}(t)) - \sum_{k=1}^{K} p_{pk}(w_{pk}(t)) \right] \\
& + \sum_{t=1}^{T} \mu(t) \left[P_r(t) - \sum_{i=1}^{I} r_{ti}(p_{ti}(t)) \right. \\
& \left. - \sum_{j=1}^{J} r_{hj}(p_{hj}(t)) - \sum_{k=1}^{K} r_{pk}(p_{pk}(t)) \right]
\end{aligned}
\tag{11}
$$

After regrouping relevant terms, individual thermal, hydro, and pumped-storage sub-problems are formed, one for each unit. A hierarchical algorithm can thus be constructed. With given multipliers, individual subproblems are solved at a low level. At a high level, the following dual problem is solved.

$$
\operatorname*{Max}_{\lambda, \mu \geq 0} \phi(\lambda, \mu), \text{ with } \phi(\lambda, \mu) \equiv \operatorname*{Min}_{p_{ti}(t), w_{hj}(t), w_{pk}(t)} L \tag{12}
$$

The resolution of thermal and hydro subproblems can be found in Ref. 4. The resolution of pump-storage power stations will be presented next, and the resolution of the dual problem in a later section.

Solving Pumped-Storage Subproblems. The pumped-storage power station subproblem is:

$$
\begin{aligned}
\min_{w_{pk}(t)} & L_{pk}, \text{ with } L_{pk} \\
& = \sum_{t=1}^{T} \{-\lambda(t) p_{pk}(w_{pk}(t)) - \mu(t) r_{pk}(p_{pk}(w_{pk}(t)))\}
\end{aligned}
\tag{13}
$$

subject to constraints of Eqs. (4), (5), (6), (7), (8), (9) and (10).

Basic Idea—Relax the Reservoir Level Dynamics. Given λ and μ, the pumped-storage subproblem is to determine the generation/pumping level at each hour so as to minimize the subproblem cost function of Eq. (13), subject to the individual constraints Eqs. (4, 5, 6, 7, 8, 9, 10). Although this cost function is stage-wise additive, the generation/pumping at hour t cannot be determined by simply minimizing the stage-wise cost function at that hour since the reservoir level dynamic constraints of Eq. (4) couple decisions across hours. The basic idea of the algorithm is then to relax Eq. (4) by using another set of multipliers. The optimal generation/pumping level at a particular hour can then be easily obtained by optimizing a single variable function, and the multipliers can be optimized at a middle level to satisfy the once relaxed dynamic constraints.

Nonlinear Approximation. Since both $p_{pk}(w_{pk}(t))$ and $r_{pk}(w_{pk}(t))$ are piece-wise linear with respect to $w_{pk}(t)$, $L_{pk}(w_{pk}(t))$ is also a piece-wise linear function of $w_{pk}(t)$.. Since the minimum point of a piece-wise linear function is at the corners or boundaries, the optimal generation (pumping) level is generally obtained at one of these points: maximum generation (pumping), minimum generation (pumping), maximum reserve, or idle. The optimal decision therefore jumps from one of these points to another as the multiplier changes. The subproblem solution oscillation makes it difficult to converge at the high-level where multiplies are updated. To overcome the difficulty, a quadratic function is used to approximate the water-power conversion $p_{pk}(w_{pk}(t))$ as depicted in Fig. 5, and $L_{pk}(w_{pk}(t))$ will be piece-wise quadratic accordingly. The optimal generation or pumping therefore no longer jumps from one corner point to another over the iterations, and the difficulties caused by solution oscillation can be avoided. To differentiate from the original linear water-power conversion, the new conversion is denoted as $\hat{\mathrm{I}}_{pk}(w_{pk}(t))$, and the new cost as $\hat{L}_{pk}(w_{pk}(t))$.

Optimal Generation/Pumping Levels. To obtain the optimal generating/pumping level at each hour, the reservoir level dynamics of Eq. (4) and the boundary constraints of Eq. (5) are first relaxed. From Eq. (4), the reservoir level at hour t can be determined based on the initial level, and water discharged or pumped as:

$$v_k(t) = V_k^0 - \sum_{n=1}^{t} w_{pk}(n) \tag{14}$$

By substituting the preceding equation into the Eqs. (5), (6) and (7), one has,

$$V_k^0 - \overline{V}_k \le \sum_{n=1}^{t} w_{pk}(n) \le V_k^0, t = 1, \ldots T-1 \tag{15}$$

and,

$$\sum_{n=1}^{T} w_{pk}(n) = V_k^0 - V_k^{\mathrm{T}} \tag{16}$$

It can be seen that the constraints of Eqs. (15) and (16) couple the decision variable $w_{pk}(n)$ among hours. By using additional sets of multipliers, β_k, γ_k and ξ_k to relax Eqs. (15) and (16), the subproblem cost function becomes:

$$\hat{L}_{pk}(\lambda, \mu, \beta_k, \gamma_k, \xi_k) = \hat{L}_{pk}(\lambda, \mu) + \sum_{t=1}^{T-1} \left\{ \beta_{\mathrm{k}}(t) \left[\sum_{n=1}^{t} w_{pk}(n) - V_k^0 \right] \right.$$
$$\left. + \gamma_k(t) \left[V_k^0 - \overline{V}_k - \sum_{n=1}^{t} w_{pk}(n) \right] \right\} + \xi_k \left[\sum_{n=1}^{T} w_{pk}(n) - (V_k^0 - V_k^{\mathrm{T}}) \right] \tag{17}$$

With given multipliers β_k, γ_k and ξ_k, Eq. (17) is a separable sum in time. To make this point clearer, define the stage-wise cost function $h_k(w_{pk}(t))$ at hour t as

$$\begin{aligned} h_k(w_{pk}(t)) = {} & -\lambda(t)\hat{p}_{pk}(w_{pk}(t)) - \mu(t) r_{pk}(\hat{p}_{pk}(w_{pk}(t))) \\ & + \left[\sum_{n=t}^{T-1} (\beta_k(n) - \gamma_k(n)) + \xi_k \right] w_{pk}(t), \\ & t = 1, 2, \ldots T-1 \end{aligned} \tag{18}$$

and

$$\begin{aligned} h_k(w_{pk}(T)) = {} & -\lambda(T)\hat{p}_{pk}(w_{pk}(T)) \\ & - \mu(T) r_{pk}(\hat{p}_{pk}(w_{pk}(T))) + \xi_k w_{pk}(T) \end{aligned} \tag{19}$$

Regrouping terms in Eq. (17) according to hours and using the defined stage-wise function $h_k(w_{pk}(t))$, one can rewrite pumped-storage subproblems as:

$$\min_{w_{pk}(t)} \hat{L}_{pk}(\lambda, \mu, \beta_k, \gamma_k, \xi_k), \text{ with } \hat{L}_{pk}(\lambda, \mu, \beta_k, \gamma_k, \xi_{\mathrm{k}})$$
$$= \sum_{t=1}^{T} h_k(w_{pk}(t)) + \sum_{t=1}^{T-1} \left\{ \gamma_k(t)(V_k^0 - \overline{V}_k) - \beta_k(t) V_k^0 \right\} - \xi_k (V_k^0 - V_k^{\mathrm{T}}) \tag{20}$$

subject to the operation range constraints of Eqs. (8), (9) and (10). In Eqs. (20), the last two terms are constants with given multipliers λ, μ, β_k, γ_k, ξ_k, and can be ignored in the minimization. It is then clear that $\hat{L}_{pk}(\lambda, \mu, \beta_k, \gamma_k, \xi_k)$ is separable in time with given multipliers λ, μ, β_k, γ_k, ξ_k. The optimal water discharged or pumped for a particular hour t can then be obtained by minimization $h_k(w_{pk}(t))$,

$$w_{pk}^*(t) = \arg \min_{w_{pk}(t)} h_k(w_{pk}(t)) \tag{21}$$

subject to the range of the operation mode.

Optimize the Multiplier for Reservoir Levels. An intermediate level is created to update the multipliers β_k, γ_k and ξ_k. Let $\hat{L}^*{}_{pk}(\lambda, \mu, \beta_k, \gamma_k, \xi_k)$ denote the optimal Lagrangian for Eq. (20). The multipliers $\beta_k(t)$, $\gamma_k(t)$

and ξ_k are updated at an intermediate level by a subgradient algorithm to maximize the Lagrangian, that is,

$$\max_{\beta_k(t),\gamma_k(t),\zeta_k} \hat{L}^*_{pk}(\lambda, \mu, \beta_k, \gamma_k, \zeta_k) \tag{22}$$

The subgradient algorithm to update $\beta_k(t)$, $\gamma_k(t)$ and ξ_k is presented next.

Solving the Dual Problems. The high-level dual problem is to update the multipliers λ and μ associated with demand and reserve requirements so as to maximize the dual function of Eq. (12). Since discrete decision variables (thermal units on or off, and pumped-storage units pumping, generating, or idle) are involved at the low level, the dual function $\phi(\gamma, \mu)$ may not be differentiable at certain points (6). Gradient does not exist at the nondifferentiable points. However, a subgradient can be obtained as follows at essentially no additional cost after all the subproblems are solved:

$$g_\lambda(t) = p_d(t) = \sum_{i=1}^{I} p_{ti}(t) - \sum_{j=1}^{J} p_{hj}(t) - \sum_{k=1}^{K} p_{tk}(t) \tag{23}$$

$$g_\mu(t) = p_r(t) - \sum_{i=1}^{I} r_{ti}(p_{ti}(t)) - \sum_{j=1}^{J} r_{hj}(p_{hj}(t)) - \sum_{k=1}^{K} r_{pk}(p_{pk}(t)) \tag{24}$$

In the above, g_λ is the subgradient of $\phi(\lambda, \mu)$ with respect to $\lambda(t)$, and g_μ is the subgradient of $\phi(\lambda, \mu)$ with respect to $\mu(t)$.

With the subgradient available, a subgradient method is used to update λ and μ as follows:

$$\lambda^{l+1}(t) = \max(0, \lambda^l(t) + \alpha^l g_\lambda(t)) \tag{25}$$

$$\mu^{l+1}(t) = \max(0, \mu^l(t) + \alpha^l g_\mu(t)) \tag{26}$$

where l is the high-level iteration index, α is the step size.

The high-level iteration terminates when the dual cost L cannot be improved, or a preset number of high-level iterations has been reached. The same subgradient algorithm is also used to update $\beta_k(t)$, $\gamma_k(t)$ and ξ_k for pumped-storage subproblems. The subgradients for these three sets of multipliers are

$$g_{\beta_k(t)} = V_k^0 - \overline{V}_k - \sum_{n=1}^{t} w_{pk}(n) \tag{27}$$

$$g_{\gamma_k(t)} = \sum_{n=1}^{t} w_{pk}(n) - V_k^0 \tag{28}$$

$$g_{\xi_k(t)} = \sum_{n=1}^{T} w_{pk}(n) - (V_k^0 - V_k^T) \tag{29}$$

respectively. The multipliers $\beta_k(t)$, $\gamma_k(t)$ and ξ_k are updated at the intermediate level.

Obtain Feasible Solutions. In solving the pumped-storage subproblems, the reservoir level dynamics and boundary constraints are relaxed, and the dual solution obtained may not be feasible. Heuristics for pump-storage units are used to adjust the pumping or generating level to obtain a feasible solution. Checking from the beginning hour to the end hour, if the reservoir level at an hour t_1 is less than zero, the "over-used" water is divided into a number of small quanta. Pumping is increased and/or generating is decreased by a quantum at selected hours with small $(\lambda(t) - \mu(\mathrm{t}))$ before t_1. However, if this adjustment causes the violation of reservoir level upper bound constraints before t_1, the hour is not selected for adjusting. Similar procedures are performed if the reservoir level is greater than the maximum at an hour.

The once relaxed system demand and reserve constraints may not be satisfied. After pumped-storage subproblem solutions are adjusted to be feasible, heuristics are then used to obtain a feasible solution that satisfies the system demand and reserve. Interested readers are referred to Ref. 4 for details.

Discussions

Literature Review. One type of method for hydrothermal scheduling with pumped-storage stations is heuristics-based, such as the one developed to solve the Tennessee Valley Authority (*TVA*) power system scheduling problem (7). The TVA system consists of coal, combustion-turbine, nuclear, hydro, and pump-storage units, and the scheduling method is summarized as follows:

- Large coal units and nuclear units are operated as base units;
- Hydro units are dispatched to peak-shave the load;
- A pumped-storage model is run to determine the generating and pumping schedule with the requirement that the reservoir level be full early Monday morning, and at the lowest by Friday or Saturday evening;
- The unit commitment model based on a dynamic-programming algorithm is then run to determine thermal generation and purchase/sale schedule.

It can be seen from this process that the scheduling of the pumped-storage unit is essentially separated from that of other units. This separation made the scheduling problem easier to solve. However, overall optimization is necessary to obtain more economical results.

The other type is optimization-based. The Lagrangian relaxation framework as described earlier belongs to this type, and has been widely used in power system scheduling (5,8, and 9). The system-wide demand and reserve constraints are relaxed by Lagrangian multipliers. A two-level structure is thus used to solve the problem. At the high-level, the Lagrangian multipliers are updated, mostly by a subgradient-type method; at the low level, the individual subproblems, including pumped-storage subproblems if there are such stations in the system, are resolved. Methods differ in the way the high-level dual problem and the pumped-storage subproblems are solved.

Pumped-storage units are modeled rather simply in the early literature. Only a constraint requiring that the total water released/pumped in a cycle equal to a specified amount is considered, and the reservoir dynamics are ignored (8). With this simplification, pumped-storage subproblems are solved by merit order dispatching. A *variable metric method* is used to solve the high-level dual problem to obtain better convergence than that of the subgradient method. The advantage of the method is its simplicity in handling pumped-storage units. However, the merit order dispatching may result in the oscillation of the pumped-storage solution as the high-level multipliers are updated, and make the high-level difficult to converge. This is why the variable metric method, instead of the subgradient method, is used to improve the convergence at the high-level. Reservoir dynamics constraints should be observed in optimization, and ignoring it may affect the scheduling quality.

In the method presented in (10), the Lagrangian multipliers are updated by a modified subgradient method; at the low level, pumped-storage subproblems are solved by dynamic programming (*DP*) with all subproblem constraints being satisfied. However, DP involves discretizing reservoir levels or generating (pumping) levels and this generally requires much more central processing unit (*CPU*) time and memory. The difference between this method and the one presented in this paper is that the reservoir boundary constraints are relaxed by Lagrangian multipliers, and DP is then used to solve the pumped-storage subproblem without discretization.

It has been shown that the high-level dual function is nondifferentiable with many “ridges,” and the subgradient method may zigzag across edges resulting in slow convergence (6). A reduced complexity bundle method (*RCBM*) is thus used to update the multipliers at the high-level to avoid the zigzag. Other bundle type methods are also used (11). Generally, better convergence at the high level can be obtained by bundle-type methods.

Future Research Topics.

Solve the Scheduling Problem with Extended Time Horizon. Power systems with pumped-storage stations are commonly scheduled in a period of one day or one week (5). To obtain information about the marginal costs beyond one week, it is required to extend the time horizon to multiple weeks or even multiple months to facilitate long-term decision-making. The sizes of the high-level dual problem and pumped-storage subproblems will be significantly increased. It is believed that more efficient optimization method is needed to solve these problems with extended time horizon in order to obtain high quality schedule within reasonable CPU time.

Handle Discrete Pumping Levels. A pumped-storage station usually has several generating/pumping units. At pumping mode, each unit may be required to pump only at its rated capacity for technical or economical reasons, resulting in a few discrete pumping levels for the entire station. Slight changes of the high-level multipliers may thus result in drastic changes of the pumping level. The high-level dual problem in this case is highly nonsmooth and has many sharp ridges. Better nondifferentiable optimization methods are therefore needed for the high-level dual problem to converge when discrete pumping levels are required.

Operate Pumped-Storage Power Station in the Deregulated Environment. A pumped-storage station is capable of smoothing the demand and contributing significant reserve, but it remains unclear how these capabilities can be best utilized in a deregulated environment where energy and ancillary services, such as reserve, are unbundled. A utility has the option of self-scheduling the station, bidding it to independent system operator (*ISO*), or self-scheduling part of its capacity and bidding the remaining part. Making operating and bidding strategies for pumped-storage units will be a challenging task in the future.

Summary

Scheduling of power system with pumped-storage stations is a difficult problem, and it is hard to obtain an optimal solution. Lagrangian relaxation has been used to relax the system-wide demand and reserve constraints and obtain a near optimal solution with quantifiable quality. Pumped-storage subproblems can also be efficiently solved by combining the Lagrangian relaxation technique and DP. The difficulty associated with the nondifferentiability of the dual function has been alleviated by using bundle-type methods to improve solution quality. A challenging research topic will be how to operate pumped-storage stations in a deregulated market to make the best utilization of its unique characteristics.

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