RELIABILITY INDICES

Quantitative measures of reliability (reliability indices) are needed for the quantitative characterization of a system or a unit operation performance. (We use the term *object* to mean either a system or a unit depending on the context.) Reliability indices must reflect the most essential operating properties of the system, be understandable from a physical viewpoint, be simple to calculate at the design stage, and be simple to check at the test and/or usage stage.

To determine a reliability index one needs to formulate a clear and unique criterion of the object failure. Because failures occur randomly in time, all reliability indices have a statistical (or probabilistic) nature and in one or another way depend on time.

MEAN TIME TO FAILURE AND MEAN TIME BETWEEN FAILURES

One of the most important reliability indices are the mean time to failure (MTTF) and mean time between failures (MTBF). The MTTF is usually used for objects that are not subjected to repair (mostly components). However, it also may be referred to a system with admissible repair, if one is interested especially in the first period of time. The MTBF is used for repairable systems. In most practical cases, it is acceptable to assume for repairable systems that the indices coincide.

If there is a sample of N random time intervals before a failure, $t_1, t_2, \ldots, t_N$, the MTBF (or MTTF) can be estimated as

$$T = \frac{1}{N}(t_1 + t_2 + \cdots + t_N)$$

If a prior distribution $F(t)$ of an operating time before failure is known, the expected value of T is defined as

$$T = \int_0^\infty t\,dF(t)$$

that is equivalent to the expression

$$T = \int_0^\infty P(t)\,dt$$

where $P(t) = 1 - F(t)$ is the probability of failure-free operation.

Although MTTF and MTBF are similar in their meaning, they are different. Considering a repairable object, one should distinguish an expected time to the first failure, from the expected time to failure, say, after the tenth repair. The simplest and most graphic explanation can be given by an example of a duplicated system with independent repair of units described by the Markov model depicted in Fig. 1. (For details, see REPAIRABLE SYSTEMS.)

Both units are assumed identical with exponential distribution of random time to failure. At the initial moment of time $t = 0$, the system has both units operational [see Fig. 1(a)]. Denote the MTTF by T_{02}. On the average, in time t^* one of system's units fails and repair has begun. At this moment the system has a single operating unit [see Fig. 1(b)]. Denote the MTTF from this state by T_{12}. Of course, a failed unit can be repaired and the system will return to the initial state [see the left side of Fig. 1(c)]. However, from this state the system might also fail. In this case, the system MTTF can be defined as $T_{02} = t^* + T_{12}$. Note that once the system has failed and then one of its units has been repaired, the system again appears in the state with a single operational unit, that is, the mean time from this moment equals T_{12}. However, $T_1 < T_2$ always.

In general, a new item is usually better than a used and repaired item, so survival time to the first failure is expected to be larger (on the average) than survival time after repair.

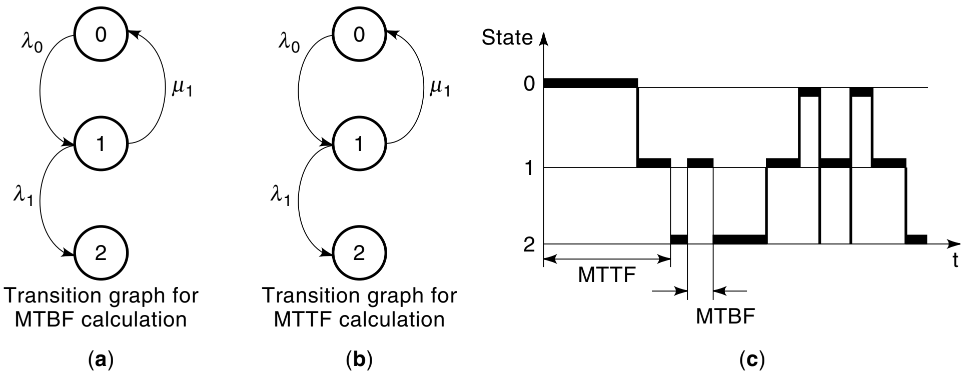

Figure 1. Transition graph and time diagram for illustration of the difference between mean time between failures (MTBF) and mean time to failure (MTTF).

(Of course there is no effect of infant mortality, see BURN-IN AND SCREENING.)

PROBABILITY OF A FAILURE-FREE OPERATION

If an object is designed for performing operations of duration t_0, a natural reliability index is the probability of a failure-free operation (PFFO). If we observed the sample mentioned above, then the statistical estimation of the PFFO can be found as

$$P(t_0) = d/N$$

where d is the number of cases for which $t_k < t_0$. If $F(t)$ is known, the probability of a successful operation is defined as

$$P(t_0) = 1 - F(t_0)$$

FAILURE RATE

The failure rate characterizes the process of failure occurrence in time. It is defined as the mean number of failures per object per unit of time. If one tests (uses) N identical objects during time t, replaces them after failure with identical ones, and observes d failures in total, then the failure rate of this object is usually estimated as

$$\lambda^* = d/Nt$$

This definition is precise for equipment characterized by the Poisson process of failures, which, in turn, corresponds to exponential distribution of time to failure. Fortunately, this definition is practically usable for complex systems consisting of a large number of units, even if those units have arbitrary distributions of time to failure. In this case, the failure rate is considered equal to the inverse value to MTBF. According to a well-known theorem of the theory of stochastic processes, superposition of a large number of failures produces an approximately Poisson process (see REPAIRABLE SYSTEMS).

INTENSITY OF FAILURE

The intensity of failure characterizes the process of reliability characteristics changing over time. Let us consider a test (use) of N objects without replacement. The intensity of failure is defined as

$$\lambda(t) = \frac{d(\Delta_t)}{N(t) \cdot \Delta_t}$$

where Δ_t is some time interval (say, from $t - \frac{1}{2}\Delta_t$ to $t + \frac{1}{2}\Delta_t$), $N(t)$ is the number of objects tested (or used) at moment t, and $d(\Delta_t)$ is the number of failures within Δ_t. That is, the intensity of failure at moment t is the expected number of failures per number of objects survived up to the moment t per unit of time. In probabilistic terms, $\lambda(t)$ is the instantaneous conditional density of a failure time distribution at moment t under the condition that the object survives time t, i.e.

$$\lambda(t) = \frac{f(t)}{P(t)}$$

Historically, this function (which is also called a *hazard rate*) appeared in demography connected to the insurance business about two centuries ago. The physical sense of this function can be explained easily in the following simple terms. If one knows the prior distribution $F(t)$ with density $f(t)$, then the conditional *elementary probability* (probability for infinitesimal interval Δ) at moment t is

$$\Pr(t, t + \Delta) = \lambda(t)\Delta + \Omega(\Delta)$$

It is the probability of the death of an individual of age t during the forthcoming time interval $(t, t + \Delta)$. This function has exactly the same sense in reliability theory when one substitutes the corresponding terms.

Reliability of real objects changes with time. Usually the first period has a higher hazard rate (in demography this is the period of infant mortality; in engineering it is called the burning-in process), and the survivors behave more or less stably during the period of *normal life,* that is, $\lambda(t)$ is constant. During the latter period, failures occur occasionally. Then, during the last period, the aging process leads to an increase of the hazard rate from, for example, wearing out or fatigue. This type of function $\lambda(t)$ is depicted in Fig. 2.

Failure rate and intensity of failure have close formal definitions, which sometimes leads to confusion. In brief terms, in reliability the failure rate is a characterization of the expected number of failures of a renewable object, whereas the intensity of failure describes the expected behavior of an individual object up to the (first) failure.

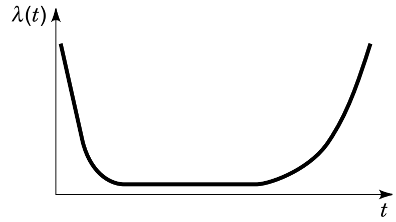

Figure 2. *U*-shaped form of function $\lambda(t)$.

MEAN REPAIR TIME

For repairable objects, an important reliability index is the mean repair time (MTR). The meaning of this index does not need any comment. Note that if we consider a repairable system consisting of nonrepairable units that are replaced in case of failure, the system MTR equals the mean time of failed unit replacement.

AVAILABILITY COEFFICIENT

Consider a system that has to work in a *waiting* regime and, at the same time, the duration of the task performance is negligibly small. In this case, a natural reliability index is the so-called availability coefficient, $K(t)$. This index is the probability that the system will be in an operating state at a specified moment t in the future. In general, availability coefficient depends on time t. (For possible types of behavior of function $K(t)$, see Fig. 3.)

With time, $K(t)$ goes to some constant value, or stationary availability coefficient K. In engineering, one usually is interested in this value, which is expressed in mathematical terms as $K = \lim_{t\to\infty} K(t)$. In engineering, K is defined as

$$K = \frac{\mathrm{MTBF}}{\mathrm{MTBF} + \mathrm{MTTR}}$$

COEFFICIENT OF INTERVAL AVAILABILITY

If a system is operating in the *waiting* regime but the duration of the system's task is not negligibly small, one speaks about the reliability coefficient, or the coefficient of interval availability. This is the probability that at a time t the system is found operating and will have not failed during t_0, the performance of the task. Denote this index by $R(t, t_0) = K(t) P(t_0|t)$.

Usually the stationary value is of interest, that is, $R(t_0) = \lim_{t\to\infty} R(t, t_0)$. This reliability index can be written in the form

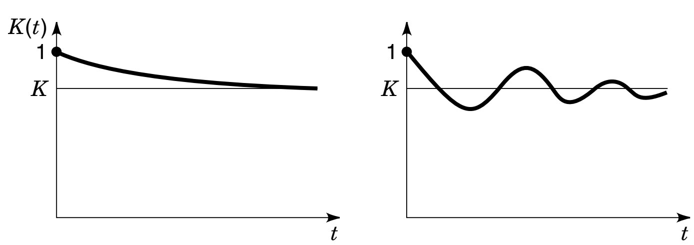

Figure 3. Two main types of function $K(t)$ behavior.

$R(t_0) = K \cdot P^*(t_0)$, where $P^*(t_0)$ is the probability that the system in the stationary process will successfully operate during additional time t_0. It is known (1,2) that for small t_0, that is, for $t_0 \ll \mathrm{MTBF}$,

$$R(t_0) = K \exp\left\{-\frac{t_0}{\mathrm{MTBF}}\right\}$$

SPECIAL INDICES

For some objects, failure cannot be defined in simple local terms; that is, no event can be characterized as the system failure. Several examples of such indices have been given without detailed discussion and with no standard names. Their quantitative evaluation is a complex mathematical problem that is omitted here (for details, see Refs. 1 and 2).

Extra Time Resource for Performance

A system has some reserve time to perform its task, that is, the interval of time Q_0, given for the performance of the system operation, is more than the time t_0 required for a successful operation. In other words, within time interval Q_0 the system will work without failures at least once longer than t_0.

Example. A computer system, performing a computational task of duration t_0, has a resource of time $Q_0 > t_0$. Negligibly short interruptions (errors) destroy the current result but the system has time to start the task from the beginning.

Collecting of Total Failure-Free Time

A system is required to accumulate some required amount of successful operating time, t_0, during some given period Q_0. If an accumulated time is smaller than t_0, the system is considered failed.

Example. The same computer system as above with a computational task of duration t_0 is divided into small subtasks. Each interruption (error) destroys only a small current portion of the current calculations. This stage can be repeated again and again.

Acceptable Idle Intervals

A system possesses the property of a *time-inertia;* that is, it is insensitive to short breakdowns.

Example. A computer system has an independent battery to protect the system from breakdowns of power supply. In this case, the computer system can continue to operate during some power supply breakdowns.

Performance Degradation from "Soft" Failures

For a complex system (for instance, a telecommunication system or computer network), definition of failure is very difficult if impossible. Each failure, except an exclusively *heavy* shutdown, leads to insignificant loss of the quality of the system's performance. In this situation, one considers average level of performance quality. In the simplest case, if probabilities of different system states, P_i, and the numerical characteristic of quality of the system performance, W_i, are known, $1 \leq i \leq n$, the average level of performance can be calculated as

$$W_{\text{System}} = \sum_{k=1}^{N} W_k P_k$$

More details about the system performance degradation caused by soft failures can be found elsewhere (1,2).

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RELIABILITY IN NETWORKS. See NETWORK RELIABILITY AND FAULT-TOLERANCE.

RELIABILITY OF POWER SYSTEMS. See POWER SYSTEM SECURITY.