RELIABILITY THEORY

Reliability theory is a relatively young branch of applied mathematics, dealing most of all with probabilistic and statistical methods with applications to physical failure analysis. Probabilistic methods can be considered to be a part of general operations research, which, in turn, represents a collection of applied mathematical methods gathered within the frame of system analysis methodology.

The first work on modern reliability theory can be traced to early 1950s. Over the next two decades, practical and theoretical work on reliability sky rocketed. A number of first-rate monographs were published in the United States and the then Soviet Union. An incomplete list of early books published in English can be found in Refs. 1–6. At the beginning, this intensive development of reliability theory was primarily stimulated by military needs. Later complex and critical civilian systems (e.g., crewed satellites, huge telecommunication networks, and nuclear power plants) used reliability analysis.

Reliability theory is the basis for reliability engineering. It includes the following main branches:

- Physical models of failures of electronic objects of different types (mainly, on a component level).
- Physical models of failures of mechanical objects of different types.
- Mathematical models of systems with different types of structures (series, redundant, network type, etc.).
- Mathematical models of different stochastic processes describing operating and renewing (repair or replacement) of objects.
- Mathematical models of software reliability.
- Stochastic simulation of the system behavior under the influence of a failure mode.
- Optimization models of redundancy and spares; preventive and regular maintenance; and technical diagnosis.
- Statistical inferences and test planning.
- Models for special tests (e.g., accelerated)

PHYSICAL MODELS OF FAILURES

Physical models of failures of electronic and mechanical objects are, probably, the most important application of reliability theory. These models allow us to improve processes of production and testing various components and equipment. Successful applications have been reported in the literature (4,7). Nevertheless, this part of reliability theory is in its embryonic stage. There are excellent research results in some specific areas, although there is no generalized theory (except possibly in the field of mechanical strength). One of the explanations of this fact is the fast pace of modern technology. The past thirty to thirty-five years have dramatically changed the face of electronics: instead of clumsy and ineffective electronic components, superminiaturized and extremely intelligent chips have appeared.

Many models of mechanical reliability (strength-loading models, wearing out, etc.) have a long history but traditionally are not included in reliability theory. (See STRESS-STRENGTH RELATIONS.)

Strictly speaking, this branch of technological research and development, dealing with physical models, is not usually included in current reliability theory.

MATHEMATICAL MODELS

Mathematical models of systems with different structures represent one of the more developed directions of reliability theory. These models present the system's main structures: series, parallel, different series-parallel and parallel-series structures, complex types of redundancy (like k-out-of-n, dynamic redundancy, etc.), two-pole and multipole networks, cold standby, and so on. (See RELIABILITY ANALYSIS OF REDUNDANT AND FAULT-TOLERANT SYSTEMS.)

In most mathematical reliability models, units are assumed to be independent, which allows us to obtain practical results for different reliability indices (5,6,8–10). Another important assumption concerns an exponential distribution of time to failure (or between failures). Many results in the closed form are obtained under this assumption. Applicability of these assumptions is discussed below.

Network structures of general form can be analyzed with Monte Carlo simulation (see MONTE CARLO SIMULATION). Such models usually are not restricted by pure reliability analysis. For instance, models of telecommunication networks include a possibility of analysis of different structure of traffic, protocols, possibility of buffering messages, time delays, and so on.

There are mathematical models analyzing the probability of connection in two-pole and multipole networks. For example, lower and upper bounds (Esary-Proschan and Litvak-Ushakov) for two-pole networks are obtained (2,8,9).

Important theoretical results were obtained for the so-called increasing/decreasing failure rate distributions (IFR and DFR). These results allow us to transfer some results obtained for models based on the assumption of exponential distribution of time between failure on models with arbitrary distributions (2).

MATHEMATICAL MODELS OF STOCHASTIC PROCESSES

Mathematical models of stochastic processes describe the behavior of different systems in time. The simplest models are based on the Poisson process, which is a point process. Intervals between neighboring events have exponential distribution in a Poisson process. For repairable systems with a complex structure, the most developed reliability models are based on Markov processes. A Markov process is a process with discrete states that last for an exponentially distributed time. Transitions between them occur in accordance with the so-called embedded Markov chain. The main feature of such models is their so-called Markov property, which means that a current behavior of the process does not depend on its entire *prehistory*. This assumption, although it seems artificial, satisfies many real-life situations (5,6,8,9). More exotic models grounded on semi-Markov processes permit one to consider a process with arbitrary distribution of time for visiting different states. It is relevant that semi-Markov reliability models are driven by pure academic interest, because there is no statistical data for practical use.

The powerful asymptotic methods of reliability analysis of the so-called "highly reliable systems" were developed in the past decade. (In such highly reliable systems, the probability of failure or the coefficient of unavailability of the system is much less than one.) These methods are based on Khinchin and Renyi limit theorems on *thinning* stochastic point processes and on the Grigelionis–Pogozhev limit theorem on superposition of stochastic processes (8).

The limit theorems on thinning state that if one excludes events from an arbitrary point process in correspondence with a Bernoulli trial, a Poisson process will be formed asymptotically in the limit (with a natural normalization procedure). For practical problems, failures of a highly reliable redundant system will appear approximately at exponentially distributed random time intervals.

The second limit theorem states that if one superimposes independent and "equally small" arbitrary stochastic point processes, a Poisson process will be formed asymptotically in the limit. For practical problems, for instance, failures of a series system consisting of a large number of units will form approximately a Poisson process.

These limit theorems opened wide the prospects for obtaining constructive results for analyzing highly reliable systems (5,8) (see REPAIRABLE SYSTEMS).

MATHEMATICAL MODELS OF SOFTWARE RELIABILITY

Mathematical modeling of software reliability is another potentially important direction. One of the main causes of modern sophisticated electronic equipment failure is *bugs* in the software. Engineers could create a perfect hardware design, but a system as a whole can be insufficient in its performance. Software has significant differences from hardware, especially where the application of reliability theory is concerned:

- Bug appearance in time depends on the system use (some bugs affect one system and do not affect another because of a different use)
- Bug appearance is not stochastic as in hardware (it again depends on the schedule of its use)
- Bug appearance in all identical software items is strictly dependent; each copy of software is an exact replica of a master copy.

All these factors clearly show that a blind application of standard reliability methods developed for hardware is not correct for software. Some models (like the reliability growth model) are appropriate, but correct reliability models of software still remain largely enigmatic. Probably, software reliability models should be based on principally new concepts.

STOCHASTIC SIMULATION

Stochastic simulation is a powerful method of analysis of structure reliability and time-dependent behavior of complex systems. This method in its modern form was formulated some 50 years ago by John von Neuman strictly for calculation purposes. Monte Carlo simulation is based on an imitation of real-system behavior on a computer using numbers generated randomly with the required properties. For such a simulation, one needs to know the system structure, the mutual operation of the system's units, algorithm for system operation, and so on, which should be given in a strict descriptive form. The algorithm of simulation allows us to avoid the use of analytical (mathematical) descriptions in the form of formulas and equations (see MONTE CARLO SIMULATION).

Briefly speaking, the procedure of Monte Carlo simulation consists of generating a sequence of system states, checking each state with respect to formulated system failure criteria, and then transiting to the next state in accordance with generated random numbers. (Each new event in the system is determined by respective random variables calculated on the basis of inverse transformation of random numbers generated by the computer.)

Simulation results, which are very similar to results from testing or using a real system (although in "compressed" time) can be processed afterward as ordinary field reliability data. The accuracy of the final result is determined by the total time of simulation (i.e., by the size of a sample of obtained data).

Highly reliable systems, where failures occur very seldom (in other words, many changes of system's states could occur between system failures), need special accelerating of the simulation process. The reader can find discussion on this special topic elsewhere (9).

There are different modifications of stochastic simulation beyond this general description, each of which is used in a particular case.

Reliability Optimization Models

Reliability optimization models are very significant, because reliability analysis should not be just a post mortem analysis. It determines the place and role of cost-effective analysis in reliability theory (see OPTIMIZATION IN DESIGN FOR RELIABILITY). Such a model should distinguish the following:

- Optimal redundancy and spares
- Optimal preventive and regular maintenance
- Optimal technical diagnosis

Optimal Redundancy

Optimal redundancy is the most important and most developed reliability tool used today. The problem, formulated in verbal terms, is as follows:

1. To reach the required reliability level of the system by means of minimum possible cost, that is,

$$\min_{1 \le k \le n} \{C(x_1, x_2, \ldots, x_n) | R(x_1, x_2, \ldots, x_n) \ge R_R\}$$

   where x_k is the number of redundant (spare) units of type k, n is the number of different groups of redundant units, $C(x_1, x_2, \ldots, x_n)$ is the system cost, $R(x_1, x_2, \ldots, x_n)$ is the system reliability, respectively, under condition that there are $(x_1, x_2, \ldots, x_n)$ redundant (spare) units in the system, and R_R is the required reliablity.

2. To reach the maximum possible reliability level under the condition that the system cost does not exceed a specified level C_S, that is,

$$\max_{1\le k\le n}\{R(x_1,x_2,\ldots,x_n)|C(x_1,x_2,\ldots,x_n)\le C_S\}$$

These models have a restricted usage in built-in redundancy because normally the number of redundant units does not exceed two or three for a group of operating units. However, the optimal allocation of spares is a very important engineering task whose successful solution can bring an essential cost reduction.

For a solution of direct and inverse problems of optimal redundancy, one uses the method of steepest descent, dynamic programming (including the Kettelle algorithm), branch and bound method, and others. The structure of optimal redundancy problems is such that both goal functions, $C(x_1, x_2, \ldots, x_n)$ and $R(x_1, x_2, \ldots, x_n)$ can be presented for practical applications in additive form:

$$C(x_1,x_2,\ldots,x_n)=\sum_{k=1}^{n}c_k x_k \text{ and}$$
$$\ln R(x_1,x_2,\ldots,x_n)=\sum_{k=1}^{n}\ln R_k(x_k)$$

This allows us to apply a simplest method–steepest descent for practical solutions. A detailed description of methodology for solving optimal redundancy problem with realistic examples has been presented (2,8,9,11).

If there are no strict cost restrictions or exact reliability requirements, there arises a cost-effectiveness trade-off problem of the Pareto optimization type. An explanation of this solution (in simple terms) is as follows. One finds the maximum possible reliability given some advance restriction on the system cost. Assume that this solution is not satisfactory. It is possible to increase an admissible system cost and see what new reliability level is achieved. The decision maker decides if it is reasonable to spend that money for reliability improvement and to take the next step in the same direction. Some discussion of the topic above can be found elsewhere (8,9).

Lastly, the optimal redundancy problem with several restrictions (simultaneously taking into account cost, weight, volume, etc.) also arises in engineering practice (2,8).

The optimal redundancy problem is very close to the optimal inventory supply problem. Indeed, it is impossible to consider spares allocation without considering refilling the stock (periodically or by request). Some problems of such a type have been considered (9).

Close problems of the so-called dynamic redundancy concerns an intermediate case between "static" optimal redundancy and optimal inventory supply. In this case the failure of a unit leads to system failure; however, the advance replacement cannot be a cause of the system failure. One chooses (in advance or on the basis of current information) moments when operating units are replaced by standby units (8).

Optimal Maintenance

The optimal maintenance problem consists of choosing the period and depth of maintenance. Maintenance might be regular and preventive. (The latter model of dynamic optimal redundancy is similar to problems of optimal maintenance.) Several mathematical models of maintenance have been considered (9,12). It seems that the main effort should be made in designing adaptive procedures of optimal maintenance.

Optimal Failure Diagnosis

Optimal failure diagnosis mathematical models of search and localization of failures are very simple and mostly illustrative.

In some sense, this problem (mathematical models of diagnosis) belongs to the past when technical diagnosis was performed manually. Program controlled technical diagnosis (built-in control) solves this problem very effectively. This direction presents a number of promising engineering solutions, although there is no collection of stable technological methods.

STATISTICAL INFERENCES AND TEST PLANNING

Collection of information about failures (location of failure, its cause, time from the previous failure, etc.) is an important phase in feedback from equipment users to equipment designers. Data about failures are processed and kept in special databases for use by designers (see STATISTICAL ANALYSIS OF RELIABILITY DATA). Practically, there are no specific statistical methods for reliability data processing. One uses standard methods of point and confidence estimation.

A special case is represented by truncated data where some observations have been interrupted before a failure may have occurred. In this case Kaplan-Meier estimates or its modification [for instance, estimate proposed by I. Pavlov and I. Ushakov (9)] are used.

A new, specific reliability approach uses confidence estimation of a system on the basis of test results of its units (7–9,13,14). This approach is very important because it allows the incorporation of unit test results into the system mathematical model and obtains the final results after system testing. This method is especially important for developing systems in the future. It should be emphasized that a special branch of statistical inferences in reliability relies on a Bayesian approach (14) (see BAYESIAN INFERENCE IN RELIABILITY).

Standard acceptance-rejection procedures are used in mass production. An important direction in statistical reliablity is test planning. It provides an early understanding about the number of units for test, the length of testing, and so on. The sequential analysis (Wald method) presents an example of a flexible acceptance–rejection procedure with a current testing procedure (9).

Highly reliable objects (e.g., modern electronic components) need a huge number of items tested for a long period of time. To avoid the enormous cost of such tests and at the same time to obtain the required results, special accelerated tests can be used. During these tests, items operate under different stresses (temperature, humidity, vibration, etc., depending on the specific task), which cause faster occurrence of failures. The main problem is to use such an acceleration of stress that the increased failure rate still preserves the mechanism of failure. Having gotten statistical results, special models based on the principles of automodality are used. More details are given elsewhere (Refs. 9 and 15; also see ACCELERATION MEASUREMENT.)

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RELIABILITY, TRANSISTOR. See POWER DEVICE RELIABILITY.