TACTILE SENSORS

Tactile sensing refers to the perception of stimuli related to the sense of touch. Tactile sensors are electromechanical devices that convert touch stimuli into electrical signals that can be processed and interpreted, typically by a computer. Tactile sensing is a research area widely studied in the fields of robotics, automation, and medicine.

The study of tactile sensing has been closely linked with our understanding of the human sense of touch. When a human makes physical contact with his or her environment, touch perception of the environment takes place in two forms—kinesthetic feedback provided by the joint and muscle receptors in our limbs, and cutaneous feedback gathered by the numerous mechanoreceptors of various types that densely populate our skin (1). Cutaneous feedback, which encompasses such stimuli as contact shape, temperature, texture, skin stretch, and slipping, is the type of tactile sensing that is typically studied in robotics. In fact, research efforts have been directed primarily toward sensing contact force and

J. Webster (ed.), *Wiley Encyclopedia of Electrical and Electronics Engineering*. Copyright © 1999 John Wiley & Sons, Inc.

shape. It is this category of tactile sensors that is the subject of this article.

A typical tactile sensor is constructed as an array of tactile sensing elements, often referred to as tactels, which are embedded in a layer of compliant material such as rubber (Fig. 1). The compliant layer protects the sensing elements from direct exposure to harsh environments and improves contact stability with its uniform and frictional surface. The overall shape of a sensor can be a rectangular block, or it can be spherical or cylindrical, depending upon where the sensor is to be mounted. The rectangular shape is useful, for example, in a parallel-jaw gripper; the spherical and cylindrical shapes are useful for instrumenting the finger tips and finger segments of a dexterous robotic hand (2).

There are two main application domains of tactile sensors: object recognition and object manipulation. In an object recognition task, a robot uses its end-effector to probe an object (a soda can, for example) the identity of which is to be determined. A tactile sensor in this case allows the robot to acquire various cues about the object while moving along the object surface. Geometric features of the object, such as curvature, edges, and vertices, can be extracted from the tactile sensory data for object discrimination. Thermoconductivity, texture, and stiffness of the object surface are some of the other cues that can be utilized. In contrast, in an object manipulation task (rotating a soda can, for example), the role of a tactile sensor is to enable the robot manipulator to determine the state of contact in terms of the contact position and contact force between the robot and the object to derive the necessary control commands and execute a manipulation plan.

What then are the desired characteristics of a tactile sensor? In a research survey conducted by Harmon in 1982 (3), a panel of university, government, and industrial researchers interested in tactile sensing were polled for an answer to this question. With very little experience to draw from, the human tactile sensing system was the primary source of information. It was postulated that a robot tactile sensor should have a spatial resolution of 1 mm to 2 mm, a force sensitivity of a few grams per tactel, a dynamic range of 1000 : 1, and a time response of 100 Hz. Design decisions of subsequent research and commercial tactile sensors have been influenced in many cases by these specifications.

The next two sections of this article deal with two important aspects of tactile sensors: touch transduction techniques, and mathematical modeling of tactile sensors. A transduction technique converts a touch stimulus to electrical signals that can subsequently be processed, and it is an issue which must be addressed before a tactile sensor can be constructed. A tactile sensor model establishes the relationship between the contact conditions and the tactile sensor output. Sensor modeling is critical for understanding the capabilities as well as the limitations of a tactile sensor.

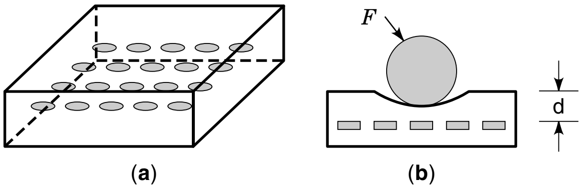

Figure 1. A planar tactile sensor: (a) Array of sensing elements embedded in a compliant material at depth d, (b) sideview of the sensor deformed by a spherical indenter applying a force $\boldsymbol{F}$.

TOUCH TRANSDUCTION TECHNIQUES

How can touch be perceived by electromechanical devices? The desire to answer this question has led to the development of a number of touch transduction techniques over the last two decades. Although the principles behind these techniques are drastically different, each and every transduction technique without exception measures one of the two physical quantities *strain* and *stress* (notions to be defined later in the following), which are related to the familiar concepts of *deformation* and internal *pressure* in an elastic body, respectively. Strain-based techniques are by far the more common of the two. As an object or *indenter* makes contact with the sensor surface, the compliant sensor body deforms, causing a redistribution of its internal pressure. The extent of the deformation depends on the location and shape of the object, the applied force, and the properties of the compliant elastic material. The array elements of a tactile sensor provide a sample of this deformation, or the resulting pressure at spatially distributed points, usually in a subsurface at depth d. Consequently, the sensor is said to produce a strain distribution or a stress distribution depending upon which one is measured. In addition, transduction techniques are typically one-dimensional (i.e., the deformation or pressure is measured only in the direction normal to the sensor surface). The behavior of a sensing element may or may not be linear with respect to the deformation or pressure. Correct measurement of the strain or stress distribution, therefore, requires a calibration of the sensing elements.

CAPACITIVE TACTILE SENSOR

A capacitive tactile sensor consists of an array of capacitors whose dielectric layer is made of an elastic material, which deforms under pressure, thereby changing the capacitance as a result. A capacitive tactile sensor is therefore based on sensing strain. Specifically, the capacitance between two conductive plates is given by

$$C = \frac{\epsilon_0 \kappa A}{d}$$

where C is the capacitance, κ is the dielectric constant, ϵ_0 is the permittivity constant, A is the area of the plates, and d is the separation distance. The capacitance increases in response to the contact force exerted between the plates.

Capacitance is measured by driving a capacitor with an alternating current (ac) signal and observing the voltage across the capacitor. It can be shown (4) that the relative change in the voltage across the capacitor is proportional to the relative change in the separation distance, that is,

$$\frac{V_s - V_o}{V_s} = \frac{\Delta d}{d_o}$$

where V_s and V_o are sensor voltage with and without load, and Δd is the relative change in separation distance with respect to the nominal distance d_o.

To create an array of tactile sensing elements, two layers of parallel electrodes—separated by the elastic dielectric layer—are placed perpendicular to each other, as shown in Fig. 2. The structure forms a grid of variable capacitors that measure the distribution of minute deformation of the elastic body. The sensor geometry is not limited to planar. Both spherical and cylindrical shapes are possible. To sample the array to obtain a strain distribution, the array is scanned sequentially, one element at a time. To select a particular element to sample, a binary decoder is used to drive one of the row electrodes with the ac drive signal, and an analog demultiplexer selects one of the column electrodes so that the capacitor between the selected row and column electrodes can be read through an operational amplifier (5).

Ultrasonic Tactile Sensor

An ultrasonic tactile sensor measures the deformation of an elastic layer based on the time-of-flight principle. Sound travels at a constant speed within a given medium [e.g., at 331 m/s in air at STP (standard temperature and pressure) and 5200 m/s in steel (6)]. As an acoustic signal travels in one medium and encounters the interface with another medium of differing elastic and inertial properties, the signal will be reflected or echoed to travel in the reverse direction, reaching an acoustic receiver. The distance between the transmitter and the interface surface (or that between the interface surface and the receiver) is given by one-half the round-trip time multiplied by the speed of sound in that medium. It should be pointed out that, if the two media share similar material properties, the echo from their interface will be of insufficient magnitude, causing this method to fail. Ultrasonic sensing has been widely used for proximity sensing for mobile robot navigation in robotics. The same principle has been applied at a microscopic level to robot tactile sensing (7).

To construct an ultrasonic tactile sensor, an array of ultrasonic transmitters is covered by an elastic medium, as shown in Fig. 3. Piezoelectric thin film such as PVDF (polyvinylidene fluoride) is commonly used for the transmitters. When an ac electric current passes through PVDF film, mechanical waves, typically of a few megahertz well beyond the audible range, are produced. The same PVDF film also serves as an ultrasonic receiver to the echo that bounces back from the surface of the elastic medium. The echo triggers a timing circuit to record the time of flight and calculate the distance. Note that the array of ultrasonic transmitters/receivers must be mounted on a rigid substrate so that they do not shift when a load is applied to the sensor surface.

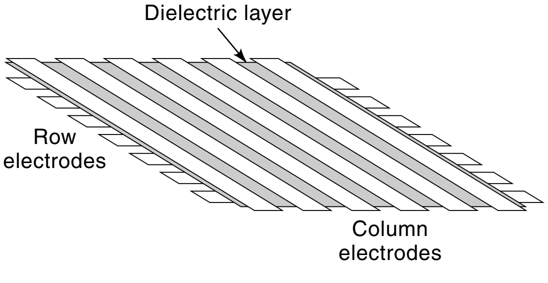

Figure 2. Structure of a planar capacitive tactile sensor with two layers of parallel electrodes separated by a compliant dielectric layer (shaded).

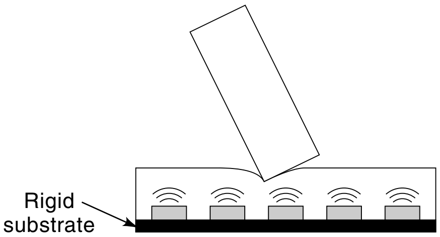

Figure 3. Sideview of a planar ultrasonic tactile sensor with an array of PVDF-based transmitters/receivers, embedded in an elastic medium. Also shown is a rectangular indenter in contact with the sensor surface.

Similar to a capacitive tactile sensor, an ultrasonic tactile sensor is based on measuring strain. One major difference, however, is that an ultrasonic tactile sensor measures the integral of strains or the total displacement of sensor surface from its nominal position, rather than strains at a subsurface as in the case of a capacitive sensor. This characteristic may appear to simplify the problem of sensing object shape, but this is the case only if one could resolve the region of contact. The region of contact, unfortunately, is not equivalent to the region of surface deformation, which in general extends indefinitely.

One important consideration in the operation of an ultrasonic tactile sensor is to avoid signal interference among the sensing elements. There are two forms of interference that can easily occur. First, the ultrasonic signal transmitted by one element generates an echo that can easily be picked up by the other elements of the array. To solve this problem, the sensor electronics must maintain sufficient temporal separation in between the sampling of adjacent sensing elements. The second form of interference is a result of attempting to reduce the number of electrodes to energize PVDF. Although one can use a single PVDF strip for an entire row of sensing elements, and one strip for an entire column of sensing elements, signal interference becomes a serious problem—especially for large arrays—because of the capacitance effect of the layered PVDF strips (7). One solution to this problem is to construct electrically insulated but acoustically coupled rows and columns of PVDF strips, and then use the rows to transmit and the columns to receive. The capacitive effect is removed as a result.

Piezoresistive Tactile Sensors

Piezoresistivity refers to the material property that the electrical resistance of the material changes when it is subject to pressure. Piezoresistive materials are therefore obvious candidates for constructing tactile sensors. Piezoresistivity can be obtained in several different ways, among which conductive elastomer, carbon felt, and force sensing resistor (FSR) are the most popular.

Conductive Elastomer. Conductive elastomer is a silicone-based rubber that contains electrically conductive particles. The overall behavior of the material is resistive with a resistance value that varies with its shape. When a blob of conductive elastomer is sandwiched between two electrodes [Fig. 4(a)], the shape of the blob varies under the applied pressure, and the resistance through the blob changes as a result, pro-

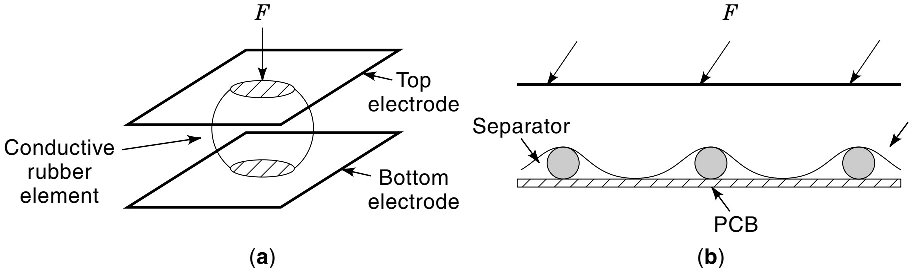

Figure 4. Principle of operation of tactile sensors based on conductive elastomer. (a) Resistance of the conductive rubber element provides an indication of the applied force ***F*** and (b) an array of such elements are formed by placing insulating separators between the conductive elastomer (ACS) and parallel electrodes (PCB).

viding information about the contact force (8). In this design, in order to produce an array of sensing sites, two layers of parallel electrodes are placed perpendicular to each other, and the intersection points are individually accessed and sampled to produce the pressure values at those discrete points.

The construction of a tactile sensor based on conductive elastomer can be simplified by employing anisotropically conductive silicone (ACS), which has the characteristic of being conductive along only one direction (9) (i.e., in the plane of a sheet of ACS, electricity conducts along x but not y). This directional characteristic is achieved by alternately concatenating conductive and nonconductive thin rubber strips so that the resistance between conductive strips is practically infinite, and along a conductive strip it is that of a resistor. In this case, ACS itself can serve as both sensor and electrodes. When a second set of parallel electrodes in the form of a flexible printed circuit board (PCB) is placed perpendicular to the ACS strips, a matrix or grid of pressure sensors is formed [Fig. 4(b)]. By placing a thin mesh of insulating material to separate the ACS and the PCB, there is no contact between the two conducting layers in the absence of a load. When a load is applied, ACS is pressed through the gaps between separators, making the two conductive layers electrically connected. The contact resistance from an ACS strip to a PCB conductive strip decreases with the area of contact, which in turn increases with the applied load.

There are two major problems associated with tactile sensors based on conductive rubber. The first problem is creep (i.e., a decrease in resistance without change in load). The second problem is hysteresis (i.e., with the conductive rubber displaying differing characteristics as the load is applied and then released). The two problems create a nonlinear and time-varying behavior in the sensor and make it difficult to calibrate the resistance value against the contact pressure.

Carbon Felt. Carbon felt or carbon fibers are pyrolyzed synthetic fibers such as rayon. When carbon fibers are compressed, the contact between the fibers increases, thus causing a decrease in the electrical resistance of the carbon fibers. Using this phenomenon, a tactile sensor can be constructed (10). Specifically, when carbon felt is sandwiched between two layers of perpendicular electrodes, a grid or an array is created and the resistance values across the carbon felt, at the crossing points between the column and row electrodes, provide an indication of the pressure distribution.

Force Sensing Resistors. A force sensing resistor (FSR) is a patented polymer, thick-film (approximately 0.5 mm) device; its resistance decreases with increasing mechanical pressure (11). Different from capacitive and conductive elastomer techniques, FSR is a stress-based sensing device. It is inexpensive to manufacture and a variety of sensor arrangements can be obtained with printed circuit technology. An array of tactile sensing elements can be created by placing two sheets, each with parallel FSR strips (Fig. 5) that represent the rows and columns of the array, at 90° with respect to each other. An addressing decoding circuit can then be employed to scan the array elements.

Because the FSR by itself has little hysteresis or creep, it is superior to conductive elastomer in this regard. The pressure-resistance relationship of FSR, however, is nonlinear. At the low operating range, FSR exhibits a switching characteristic. In mid-range, FSR is log-log (i.e., the logarithm of the resistance is inversely proportional to the logarithm of the applied pressure). At the high operating range, FSR saturates; FSR is therefore not considered a good continuous force sensing device, but more suited to pressure thresholding applications.

Optical Techniques

Optical techniques extract information about contact through sensing light interference caused by the contact. Optical fibers are an obvious choice. An array structure can be created in which rows and columns are parallel fibers (12). If the rows and columns are separated by compliant spacers, then the distance between the rows and columns varies with the load applied to the array. If the rows and the columns are not visually occluded from each other and if the surfaces of the rows and columns where they intersect are roughened to allow light to leave or enter, then by passing light through the rows and sensing the light at the columns the distance between a row and a column can be measured by calibrating this distance with respect to the light intensity.

It is also possible to sense change in pressure by a photoemitter and detector pair, an idea that was successfully used in an early commercial tactile sensor manufactured by the Lord Corporation (13). An array of photoemitter/detector

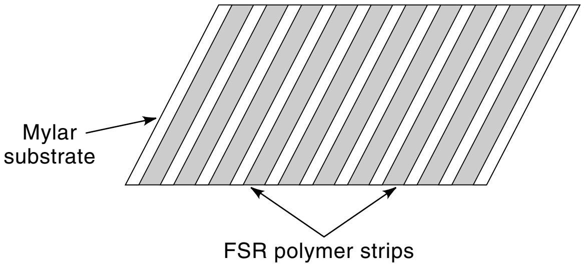

Figure 5. Parallel strips of an FSR sheet. Two such sheets are placed perpendicular to each other to form an array of sensing elements.

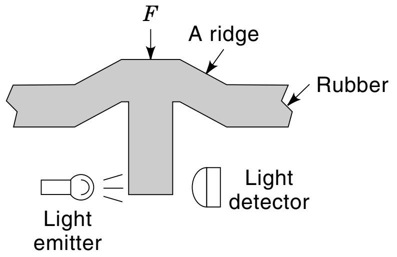

Figure 6. A tactile sensor element based on a photo-emitter/detector pair where the light intensity at the detector depends on the amount of optical occlusion, which in turn depends on the applied force ***F***.

pairs [light-emitting diodes (LEDs) and phototransistors] are mounted in a plane parallel to the sensor surface, which is made of rubber and molded with ridges to concentrate loading forces. Each photoemitter and detector pair forms a beam breaker, to be occluded by protrusions that extend downwards from the underside of the rubber surface (Fig. 6). The extent of occlusion depends on the load applied at the sensor surface, and is measured by the change in light intensity.

Another successful design of an optics-based tactile sensor makes use of an optical waveguide, the shape of which can be planar, cylindrical, or hemispherical. When light is injected into the waveguide through its edge, and if the index of light refraction of the waveguide is sufficiently larger than that of the medium outside the waveguide, light is completely confined to the waveguide. This phenomenon is known as *total internal reflection* (6). Total internal reflection occurs, for example, if the waveguide is made of glass and is either planar or curved, with a sufficiently large radius of curvature, and if the medium external to the waveguide is air. When an object makes contact with the waveguide, the condition for total internal reflection is violated and light is reflected off the object, thereby escaping the waveguide. The location as well as the shape of contact can be determined by analyzing the image of the reflected light, gathered by an array of photodiodes (14), a charge-coupled device (CCD) camera (15), or a position sensitive device (PSD) (Fig. 7).

SENSOR MODELING AND DATA ANALYSIS

The tactile sensors described in the foregoing generate outputs that depend on contact conditions. Accurate extraction of the contact conditions requires an understanding of the process by which the sensor outputs are produced. Contact conditions are typically parameterized by such variables as the local shape of the contacting object (point, line, spherical, cylindrical, etc.), the location of contact, and the forces and moments exerted through the contact on the sensor surface.

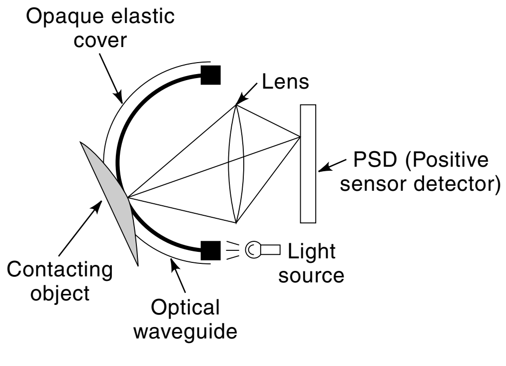

Figure 7. Principle of operation of tactile sensors based on optical waveguide. Light enters the waveguide at bottom, deflected at the point of contact (shaped arc), and sensed by the PSD.

There are two basic steps associated with the analysis of tactile sensory data: forward modeling, and inverse modeling. A forward model of a tactile sensor derives the expected sensor output, in terms of a strain or stress distribution, from a set of known contact parameters. The inverse model of a tactile sensor computes the contact parameters from the tactile sensor output. Inverse modeling is a substantially harder problem, but is required for a tactile sensor to be useful.

Problem Formulation

The problem of tactile sensory data analysis can be generally described by Fig. 1. An object, typically considered to be rigid, makes contact with the tactile sensor surface with a contact force vector **F**, consisting of three forces and three moments. The number of nonzero components of the force vector depends on the shape of the object and frictional characteristics of the sensor surface and the object. For a frictionless point contact, for example, only normal force can be applied; contact between two frictional planar surfaces allow all six force components to be exerted. The contact force causes the elastic body of the tactile sensor to deform, and this is measured by the array of sensing elements embedded within the elastic body at discrete locations in the subsurface at depth d.

One can view the elastic sensor body as a mechanical filter with its input as the set of contact parameters and the output of which is what the sensing elements measure. In the attempt to model the physics of this filtering process, researchers have largely depended on the theory of elasticity and solid mechanics, an engineering discipline which studies how an elastic body deforms and how its internal force is distributed when the body is subject to external forces.

Strain and stress are the two fundamental concepts with which an elasticity problem is formulated. Imagine an infinitesimally small cube (called a differential element) within an elastic body. When the elastic body is subject to external force, this cube will experience minute displacement from its original location and force will be applied to its six faces. The minute displacement can be characterized with a strain tensor of six independent components: three normal strains ϵ_{xx}, ϵ_{yy}, and ϵ_{zz}, and three shear strains ϵ_{xy}, ϵ_{xz}, and ϵ_{yz}. A normal strain represents the displacement per unit length of one edge of the little cube, along that edge. A shear strain, on the other hand, measures change in the angle between two edges, which are at 90° before deformation. Shear strains are extremely difficult to measure, and thus will not be considered further in this article. Note that strains are dimensionless quantities.

Associated with each face of the infinitesimal cube are three stresses: one normal stress σ_{xx} (σ_{yy} and σ_{zz} for the other two faces), and two shear stresses, σ_{xy} and σ_{xy}. The normal stress is the force per unit area applied normally to the face, and the two shear stresses are the two tangential forces in the plane of the face per unit area. A stress therefore has the dimension of force per unit area. Due to symmetry, there exists only a total of three independent normal stresses and three shear stresses to compose the stress tensor of six components.

Obviously, stress and strain distributions of an elastic medium are related to each other. This relationship is nonlinear. However, under the small-strain assumption (16), this relationship in general becomes linear and is known as the generalized Hooke's law, which involves a large number of constants related to the material properties of the medium. By further assuming that the material is isotropic and homogeneous, the relationship between stress and strain can be defined by only two constants: Young's modulus E and Poisson's ratio ν; this relationship in a matrix form is given by (17):

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{xy} \\ \epsilon_{xz} \\ \epsilon_{yz} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix}$$

Almost exclusively, all tactile sensing elements measure either the normal strain or the normal stress along the direction normal to the surface of the sensor surface (i.e., σ_{zz} or ϵ_{zz}). Multidimensional tactile sensing elements exist, but have not reached a level of maturity for practical use. Most existing techniques for tactile sensory data analysis, therefore, assume a tactile sensing array that provides a distribution of *one* normal strain or *one* normal stress, and that this distribution is the sole source of information available from which to derive the contact parameters.

The difficulty with modeling an elastic medium arises from the lack of simple mathematical tools that describe the manner in which an elastic medium reacts to external forces. If one assumes the elastic medium to be linear and isotropic—two properties possessed by most compliant materials used for constructing tactile sensors—the stress and strain within the elastic body can be characterized only by second-order linear partial differential equations that define the state of equilibrium in terms of the components of stress and strain. A forward tactile sensor model requires solving these equations for a subsurface stress/strain distribution function; an inverse sensor model must in turn solve the distribution function for the contact parameters. Unfortunately, even for forward modeling, the complete analytical solution of the differential equations has not yet been obtained, except for a few cases in which simplifying assumptions are possible. The general case can be approached only numerically using such methods as the finite element method (FEM).

Closed-Form Analytical Solutions

As mentioned, very few contact problems can be solved in a closed form, and so very few analytical forward sensor models can be established. For those contact problems that can be solved, various simplifying assumptions must be made. Hence, we will first briefly explain these assumptions, and then describe the results of representative cases (shown in Fig. 8), for which a closed-form solution can be found by making some of these assumptions.

Common Simplifications in Modeling Tactile Sensors. Table 1 summarizes the common assumptions made in modeling tactile sensors.

Representative Cases with Closed-Form Solutions. As previously mentioned, almost all tactile sensors measure the normal stress or normal strain in the direction perpendicular to the sensor surface. Consequently, we provide below only expressions for that normal stress or strain. Derivations of these expressions can be found in Ref. 18 unless otherwise cited. An explanation of their use in robot tactile sensing can be found in Ref. 19.

Point Indenting a Plane. Figure 8(a) shows the contact problem of a point indenter applying a force with a normal and a tangential component, P and Q, respectively, on an elastic half-space. If Q_x and Q_y are the two components of Q along x-axis and y-axis, subsurface stress and strain distributions are given by:

$$\sigma_{zz} = -\frac{3z^2}{2\pi r^5}(Pz + Q_x x + Q_y y) \tag{1}$$

$$\epsilon_{zz} = -\frac{3(Pz + Q_x x + Q_y y)}{2\pi E r^5}\left(z^2 - \frac{x^2 + y^2}{2}\right) \tag{2}$$

where E is the Young's modulus of the elastic material and $r = (x^2 + y^2 + z^2)^{1/2}$ is the radial distance from the point of

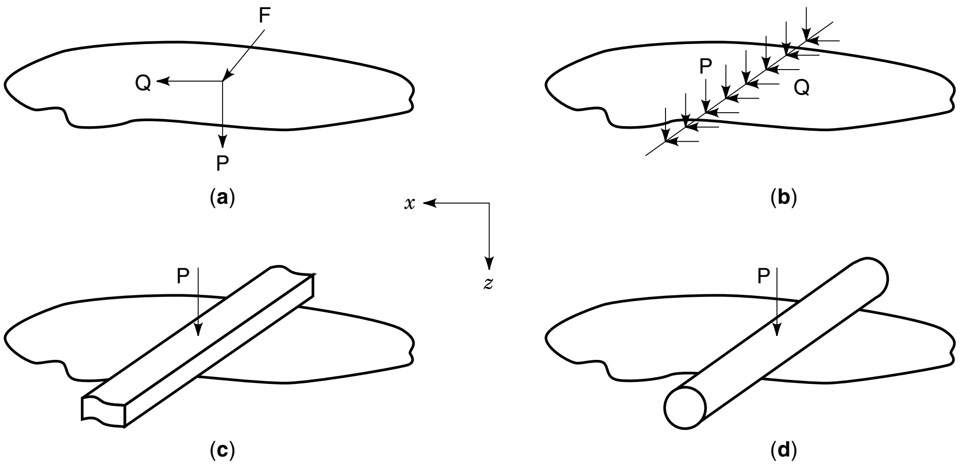

Figure 8. Representative contact problems with closed-form solutions: (a) point load $\boldsymbol{F}$ with normal component P and tangential component Q; (b) line load with normal component P and tangential component Q, both assumed constant along the direction of the line; (c) plat block applying only a normal force P; and (d) cylinder applying only a normal force P, all on an elastic half-space.

Table 1. Common Simplifications in Modeling Tactile Sensors

Rigid indenter	The object making contact with the sensor surface is rigid (i.e., it experiences negligible deformation due to contact)
Homogeneous/isotropic medium	Material properties do not vary throughout the elastic medium, nor do they vary with directions in which the properties are measured
Point/line indenter	The area of contact is small in comparison to the elastic medium such that contact forces are applied either at a point or through a line (or edge)
Elastic half-space	The elastic medium extends semi-infinitely in the radial direction away from contact surface
Plane strain	External forces are confined to act in the direction normal to a body that is essentially a cylinder, with one dimension being much larger than the other two; the strain along the large dimension is therefore zero
Axis symmetry	The elastic body is symmetric with respect to some axis such that the stress and strain distribution possesses certain symmetric properties as well
Hertz contact	The sensor and object are ellipsoidal locally, and the contact is frictionless so that only normal forces can be applied through the area of contact
Incompressibility	The elastic medium covering the sensing elements is incompressible (i.e., its total volume is constant under external body forces, so that the Poisson's ratio $\nu = 0.5$

force application to the point of interest within the elastic body.

Line Indenting a Plane. For the case of a rigid line or edge indenter in frictional contact with a planar elastic half-space, as shown in Fig. 1(b), if the line is assumed to be infinitely long, the plane strain assumption holds, and the normal stress and strain along z within the elastic half-space are given by:

$$\sigma_{zz} = -\frac{2z^2(Pz + Qx)}{\pi r^4}$$

$$\epsilon_{zz} = -\frac{2(Pz + Qx)}{\pi E r^4}\left(z^2 - \frac{x^2}{2}\right)$$

where P and Q are the normal and tangential force applied by the line *per unit length* along the line.

Flat Block Indenting a Plane. When a flat block of width $2a$ makes frictionless contact with an elastic half-space, as shown in Fig. 1(c), if P is the normal force applied by the block, the surface pressure distribution, which is usually called *surface traction* in the literature, is given by:

$$p(x) = \begin{cases} -\dfrac{P}{\pi}\dfrac{1}{\sqrt{a^2 - x^2}} & -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

from which one can derive the subsurface stress and strain distribution in polar coordinates as (20):

$$\sigma_{zz}(r,\theta) = -\frac{P}{\pi r_{12}}\left(\cos\theta_{12} + \frac{r^2}{r_{12}^2}\cos\theta\cos(\theta - 3\theta_{12})\right)$$

$$\epsilon_{zz}(r,\theta) = -\frac{P}{2\pi E r_{12}}\left(\cos\theta_{12} + \frac{3r^2}{r_{12}^2}\cos\theta\cos(\theta - 3\theta_{12})\right)$$

where $r_{12} = (r_1 r_2)^{1/2}$, and $\theta_{12} = \frac{1}{2}(\theta_1 + \theta_2)$. r_1, r_2, θ_1, and θ_2 are functions of a, r and θ, as shown in Fig. 9.

Cylinder Indenting a Plane. A long rigid cylinder of radius R applying a normal force P on an elastic half-space, shown in Fig. 8(d), results in the following surface traction:

$$p(x) = \begin{cases} -\dfrac{2P}{\pi a^2}(a^2 - x^2)^{1/2} & -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

where a is the half-width of contact between the cylinder and the elastic half-space, and it is computed by

$$a = \left(\frac{3RP}{\pi E}\right)^{1/2}$$

Subsurface normal stress distribution along the z-axis in polar coordinates can be derived as (21):

$$\sigma_{zz}(r,\theta) = -\frac{P}{\pi a^2}\left(r_{12}\cos\theta_{12} + \frac{2r^2}{r_{12}}\cos\theta\cos(\theta - \theta_{12}) - 3r\cos\theta\right)$$

where $r_{12} = (r_1 r_2)^{1/2}$, $\theta_{12} = \frac{1}{2}(\theta_1 + \theta_2)$, and r_1, r_2, θ_1, and θ_2 are functions of r, θ, and a with analogous interpretations to those in Fig. 9.

Numerical Solutions

When a closed-form solution does not exist for a contact case, one resorts to numerical solutions. There are two approaches one can take in this case. First, by decomposing a complex case of contact into a collection of simple contacts for which there are known solutions, the subsurface stress distribution can be obtained by superposing the contributions of the simple contacts. For this purpose, analysis of the stress is often preferred because in theory the law of superposition does not hold for strain but does for stress, although strain computation by superposition can be acceptable for engineering prac-

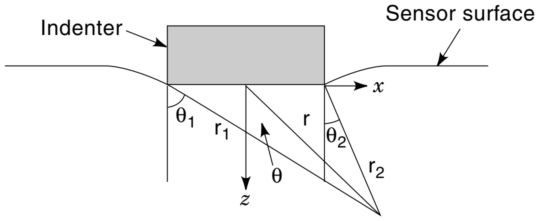

Figure 9. A flat rectangular indenter in contact with an elastic sensor surface. Strain or stress at any point of interest can be expressed as a function of variables r, r_1, r_2, θ, θ_1, and θ_2.

tice (4). The second and more general approach is the finite element method (FEM) by which a global, intractable contact problem is divided into local contact problems that can be solved numerically.

Numerical Integration. Solution by numerical integration consists of two steps. First, surface stress or strain distribution is obtained from the geometries of the object and sensor, the force applied on the object, and the material properties of the elastic sensor medium. Second, using the principle of superposition, the subsurface strain or stress distribution is obtained by considering the surface distribution as a set of concentrated point or line loads, and numerically integrating their effects at a subsurface with the closed-form solutions for point or line contact. Obviously, the numerical integration method works only if surface traction can be derived, and this is not possible for general contacts. Additionally, the method is limited by the assumptions, such as the elastic half-space, homogeneous and isotropic material properties, etc., made by the closed-form solutions.

Sphere Indenting an Elastic Half-Space. When a rigid spherical indenter of radius R makes contact with an elastic half-space (planar), the radius of the circle of contact is given by (18)

$$a = \left(\frac{9PR}{16E}\right)^{1/3}$$

where P is the normal component of the force vector. The surface tractions in normal and tangential directions in polar coordinates are given by

$$\sigma_{zz}(r) = \frac{3P(a^2 - r^2)^{1/2}}{2\pi a^3} \quad \sigma_{xx}(r) = \frac{3Q(a^2 - r^2)^{-1/2}}{2\pi a} \tag{3}$$

where Q is the tangential component of the force vector. Once again, by choosing the reference frame properly, one of the tangential forces becomes zero. One can then obtain the subsurface stress or strain distribution of the elastic medium by integrating the effects of Eq. (3) using the stress function for the point contact Eq. (1) or Eq. (2) (22).

Cylinder Indenting a Cylinder. When two cylindrical bodies make contact at an angle, as is the case when a finger of a dextrous robot hand makes contact with a cylindrical object, an approximate solution can be obtained by first finding the area of contact and surface traction, and then numerically integrating the effect of surface pressure by considering it as a collection of point loads. In this case, Hertz contact is assumed in which only normal force is exerted through the contact. The shape of the contact area is known to be an ellipse with its major and minor axes a and b dependent on the radii of curvature of the object and the sensor, the material constants of the sensor surface, contact force, and the angle between the axes of the two cylindrical bodies. Within the elliptical area of contact, surface traction is given by (18)

$$p(x, y) = \frac{3P}{2\pi ab}\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} \tag{4}$$

From Eq. (4), subsurface strain or stress can then be calculated with the stress or strain function for point contact, Eqs. (1) and (2) (23).

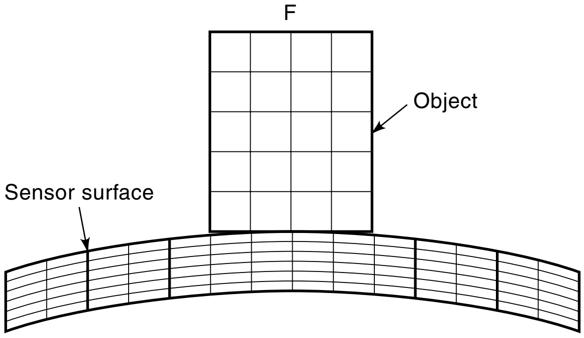

Figure 10. An example FEM sensor model of a rectangular indenter on a spherical shell (sideview). The rectangular indenter consists of 20 elements (five rows and four columns) and the sensor consists of 84 elements (six rows and 14 columns).

Finite Element Method. The finite element method offers an alternative to the study of contact problems. With FEM, the elastic body of interest is spatially divided into small elements and rather than attempting to obtain an analytical solution for the entire elastic body, a local solution is sought for each element by fitting a polynomial to the differential equations that must be satisfied by the element. The strength of FEM comes from the conversion of partial differential equations to linear algebraic equations that takes place when a polynomial is substituted into the differential equations.

A significant advantage of FEM in modeling tactile sensors is its generality. Most of the assumptions required by the closed-form solutions, most notably the elastic half-space and plane strain assumptions, are no longer necessary. Arbitrary object and sensor geometry can be handled in principle, and a wide range of loading conditions can be modeled (24,25). The main disadvantage of FEM is that the solution requires each contact problem to be described numerically. As a result, the contact parameter space must be discretized and each case solved separately. This not only is a time-consuming process, but also produces results that are inconvenient to store and manipulate.

Shown in Fig. 10 is an example of an FEM model where a rectangular peg makes contact with a spherical shell. The sensor layer has a finite thickness and is divided into six rows and 14 columns, for a total of 84 elements. The rectangular object is divided into 20 elements. Tactile sensing elements are assumed to be embedded in the elastic sensor layer. An FEM sensor model of this kind can be created with any number of commercially available FEM packages, such as ALGOR or ANSYS, and stress and strain distribution within the elastic layer can then be solved.

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Reading List

For advanced readers of tactile sensors, *Robot Tactile Sensing,* written by R. A. Russell (Prentice-Hall, 1990) (26), offers a comprehensive description of the transduction techniques for robot tactile sensing. *Contact Mechanics* by K. L. Johnson (Cambridge University Press, 1985) (18) is a valuable resource book on the subject of elasticity and offers solutions to many problems of strain and stress analysis. Finally, *Advanced Tactile Sensing for Robotics,* edited by H. R. Nicholls (World Scientific, 1992) (27), contains a series of articles from many leading authorities of tactile sensing research on a variety of topics, including transduction techniques, tactile sensor modeling, sensor data processing, human physiology pertinent touch and touch-based perception, and robotics applications of tactile sensing.

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TALKING MACHINES. See Speech synthesis.

TAPE RECORDERS. See Magnetic tape equipment.

TAPE RECORDERS, MAGNETIC. See Magnetic tape equipment; Magnetic tape recording.

TAPES, MAGNETIC. See Magnetic tapes.

TARGET ACQUISITION METHODS. See Infrared imaging.

TARGET DETECTION FOR RADAR. See Radar applications.

TARGET RECOGNITION, RADAR. See Radar target recognition.

TARGET RECOGNITION, SONAR. See Sonar target recognition.

TARGET SCATTERING. See Radar cross-section.